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THE

PHYSICAL REVIEW.

THE LEAD CELL.

By B. E. Moore.

Part I.

Theoretical.

We owe the discovery of the lead cell to Planté. The cell has undergone many practical variations, but the essential features have remained about the same. To Planté is also due the first explanation of the action of the cell. He attributed its source of energy to occluded gases. He recognized the formation of sulphate, but regarded it as a secondary phenomenon. Occlusion — a phenomenon very familiar in platinum and palladium electrodes — had been impressed strongly upon him, and the evolution of gases seemed to favor that opinion. Had this phenomenon not been very strongly impressed upon him, it seems probable that Planté might have given us the lead sulphate theory. The occlusion theory satisfied most of the scientific world for a long time. Gladstone and Tribe made a chemical study of the cell in 1882,[1] and showed that the formation of lead sulphate from lead peroxide was the principal source of the energy. Views once accredited are hard to eradicate, and the new intruder did not supplant the orthodox view without a protest. The strongest opponent of the new view was, perhaps, Dr. Oliver J. Lodge.[2]

[1] Electrician, Vol. VIII., p. 309, and subsequent numbers.

[2] Electrician, Vol. IX., p. 92, and subsequent numbers.

Other experiments followed, which showed that the occluded gases could only yield a minute quantity of energy, and the principal reaction of Gladstone and Tribe has come to be generally accepted. The Kelvin law, which demands the equivalence of heat and electrical energy, has added confidence to the chemical evidence. This treatment is a thermodynamical one, and some assert that it affords the strongest support of the sulphate theory. This law of Kelvin and v. Helmholtz was, however, shown later by v. Helmholtz to be wrong, and has been supplanted by the later v. Helmholtz equation. In this particular case it happens that the Kelvin law nearly expresses the facts. The right formula, however, can as easily be used, and the greater error of the equivalence of heat energy of a chemical reaction, and electrical energy of a similar reaction, would not be continually thrusting itself upon us.

The ideas of the new physical and electrochemistry met, at first, *much severer* attacks than the work of Gladstone and Tribe.

Discussion ought to bring out the truth, but in this case healthy and pertinent discussions soon waned, and then ceased. The adherents of the new ideas pushed forward with research and investigation, putting old facts in new light, and opening up entirely new fields, until at last the triumph is fully recognized, and a new theoretical chemistry is springing up in all of our better institutions. It would, therefore, seem desirable to have the lead storage cell explained upon the present hypotheses of electrochemistry. Le Blanc[1] has devoted a few words to the theory of the lead cell, and writes from the above standpoint. Very little experimental work bearing upon the theory of the cell has appeared in the last few years. The most recent contributions upon the theory, besides the work of Le Blanc, have been by Wade[2] and Barnett.[3] Both of these writers give an excellent historic review of the subject. Both give good discussions of the data at hand. Both ignore the recent developments in electrochemistry. Barnett even writes thermochemical equations which a decade ago

[1] Lehrbuch der Electrochemie, p. 222.
[2] Electrician, Vol. XXXIII., p. 603, and subsequent numbers.
[3] Electrical World, Vol. XXVII., pp. 403, 455, 480.

might have been accepted unhesitatingly, but to-day would be questioned by the very conservative.

The theory of Le Blanc will here be enlarged upon, and certain experiments will be given which have been suggested by the theory. The fundamental principles of Le Blanc are the theory of free ions, solution tension, and osmotic pressure, — principles, indeed, upon which rests the Nernst theory of electromotive force. Mathematic treatment, however, did not wait until experiment had matured these ideas, but appeared almost contemporaneously with them. The work of Max Planck is especially commendable. His treatment is based upon the second law of thermodynamics. His method can be applied to the storage cell. This method, *Industries* in 1891 maintained, must be applied for a proper understanding of the subject. This method of treatment seems more satisfactory to many persons, and agrees fully with the Nernst theory.

From the v. Helmholtz formula under assumption of a zero temperature coefficient, assuming the reaction,

$$PbO_2 + \text{water} + H_2SO_4 + Pb = 2\,PbSO_4 + \text{water} + 87{,}000 \text{ calories},$$

we have a voltage of 1.885. This is based upon the investigations of Streintz,[1] and agrees well enough with the value of 1.9 volts found with dilute acid. As before mentioned, these thermochemical reasons are confirmed by the chemical researches of Gladstone and Tribe. Observations upon electromotive force, with varying concentration of acid, have been carried on by Gladstone and Hibbard.[2] The observed and calculated electromotive force differs seldom more than one per cent, which would indicate in all cases a small temperature coefficient in the v. Helmholtz equation. Gladstone and Hibbard's thermochemical calculations are very abbreviated, and with such calculations grave doubts frequently arise. It would seem that a determination of the electrical output and the temperature coefficient of the cell would be a more satisfactory method of determining the "heat-toning" in such cases. However, the latter factor is difficult to determine. The heating

[1] Wien. Akad. Ber., 103, Jan., 1894.
[2] Electrician, Vol. XXIX., pp. 67, 95, 119.

effects noted by Professor Ayrton during the charging process, and the cooling of the cell noted by Dr. Duncan during discharge are good indications of a very appreciable temperature coefficient.

We will assume the battery charged and ready for use. In this condition the positive plate is covered with lead peroxide and the negative plate with spongy lead. The solution consists of sulphuric acid. For equilibrium, when solids and liquids are in contact, particles of the one must be contained within the other. At one electrode we should have lead peroxide in the solution; at the other we should have spongy lead particles in solution. Thermodynamically considered when such a substance as PbO_2 passes into solution, we can have equilibrium only upon partial dissociation of the substance. For the substance in question this follows under the formation of tetravalent $\overset{++}{\underset{++}{Pb}}$ ions and $(\overset{-}{OH})_4$ ions. The product of the concentration of the ions in a definite volume being constant, and the number of the $\overset{-}{OH}$ ions being determined by the dissociation of H_2O, the concentration of tetravalent $\overset{++}{\underset{++}{Pb}}$ ions in solution is the fourth root of the dissociated water molecules. The number of lead ions would therefore be extremely small. A theory resting upon such small quantities for carrying large currents would seem to be weak. However, in current conduction these products need penetrate only the first layers of the solution, and could, therefore, be easily supplied by the peroxide. The tetravalent $\overset{++}{\underset{++}{Pb}}$ ions pass over into bivalent $\overset{++}{Pb}$ ions during discharge, and *this is the principal source of the electromotive force of the lead cell.* The product of the concentration of the bivalent $\overset{++}{Pb}$ ions and the $\overset{--}{SO_4}$ ions, though having a larger value than the $\overset{++}{\underset{++}{Pb}}$ and $(\overset{-}{OH})_4$, is still small. As a result they combine, forming insoluble lead sulphate, which precipitates upon the electrode. This process follows under abstraction of sulphuric acid and formation of water. At the anode the bivalent $\overset{++}{Pb}$ ions unite with $\overset{--}{SO_4}$ ions, and the liberated $\overset{+}{H}$ ions migrate through the solution. This process must follow with no very appreciable potential difference at the electrode. When the cell is discharged bivalent

$\overset{++}{Pb}$ ions and the corresponding $\overset{--}{SO_4}$ ions are formed at each electrode.

In the charging process the bivalent $\overset{++}{Pb}$ ions go over into tetravalent $\overset{++}{Pb}$ ions at the positive electrode, giving $\overset{++}{Pb}$ and $(\overset{-}{OH})_4$ ions. The low solubility of this product yields PbO_2 at the electrode. Since the solution is already saturated with $\overset{++}{H_2}$ and $\overset{--}{SO_4}$ ions, the ions liberated form H_2SO_4, and the remaining $\overset{++}{H_2}$ ions wander through the solution in the direction of the current to appear ultimately at the other electrode.

At the negative electrode the $\overset{++}{Pb}$ and $\overset{--}{SO_4}$ ions yield spongy lead at the plate and form sulphuric acid with the oncoming $\overset{++}{H_2}$ ions.

The process, thus far considered, gives the formation of sulphuric acid at both electrodes, and the abstraction of water at the electrode where water had been formed in the discharging process. The process is made more complicated by the more rapid rate of migration of the $\overset{+}{H}$ ions. Conduction through the solution is thus made to depend almost entirely upon the $\overset{++}{H_2}$ and $\overset{--}{SO_4}$ ions. Le Blanc states that ions take part in current conduction in proportion to their relative concentration, but at the electrode the easiest liberation product is set free. While lead and the peroxide may exist in the stratæ next to the electrode, there would scarcely be a trace in the body of the electrolyte.

If the theory were dropped at this point it would offer no suggestions as to the electrical behavior of the cell. To gain such insight, we must make use of thermodynamics or the Nernst theory of electromotive force. Upon the latter theory, the electromotive force of a cell may be expressed by the equation

$$E = \frac{RT}{2C_0}\left(\log_e \frac{P_+}{p_+} - \log_e \frac{P_-}{p_-}\right),$$

where P and p represent solution tension of the electrode substance and osmotic pressure of the ions respectively, R the gas constant, T absolute temperature, C_0 the current generated by the liberation of one gram-equivalent of any substance, and E the electromotive force. The + and − signs refer to the elec-

trodes. Applying this formula to the lead cell, we see at once that the equation becomes one of four variables. The quantities P change, since in the charging or discharging process the character of the electrode is changing. The magnitude of p must also change. In the discharging process the bivalent $\overset{++}{Pb}$ rapidly replaces the tetravalent $\overset{++}{\underset{++}{Pb}}$. The solubility of the former is much greater than that of the latter and an increase in the value of p occurs. The reverse occurs upon charging the cell. The precipitation of a salt from solution, or the formation of a solution, occupies a certain amount of time; from which it follows, that a rapid increase in p would occur during the first few moments of a discharge, and a corresponding decrease during the charging process. Equilibrium is obtained when the current removes ions from or carries them into the solution, at the same rate at which the substance is dissolved or precipitated. This action of the current produces what one may call undersaturation and supersaturation. This action lowers the efficiency of the cell, and the stronger the current the lower the efficiency. Although there are four variables, the latter two are of little moment, and they vary in the same way as the first two. The effect upon the electromotive force of any change in the magnitude of P or p is noticeable by a simple inspection of the formula. Beside the initially rapid changes in P and p bringing about a new condition of equilibrium, there would be a slower change in the material of the electrode, and consequently slow changes in P and p.

Another series may arise in practice, which is designated as a concentration series. The following is an example: —

$$\mathrm{Ag} \mid \text{n. } \mathrm{AgNO_3} \mid \tfrac{1}{10} \text{ n. } \mathrm{AgNO_3} \mid \mathrm{Ag}.$$

The terminals are of the same material, and the only electromotive force possible arises at the contact of the solutions of different concentrations. The electromotive force of this series is expressed by

$$\frac{RT}{C_0} \log_e \frac{p}{p_1}, \text{ or } \frac{RT}{C_0} \log_e \frac{c}{c_1}.$$

c and c_1 are the concentrations of the solutions corresponding to the osmotic pressures p and p_1. The electromotive force of the

above series is evidently 0.029 volt. The character of the series would not be changed were it written in the form

$$Ag \mid n.\ AgNO_3 \mid 2\ n.\ AgNO_3 \mid \tfrac{1}{10}\ n.\ AgNO_3 \mid Ag.$$

In a two-liquid cell, a concentration series generally exists. In a storage cell at rest, there should be no trace of such a series; but in a storage cell in action, a change in the concentration at the electrodes occurs, and that very rapidly. The amount of change is dependent upon the strength of the current. We then have a concentration series of the last form, *i.e.* one with an intervening solution. The direction of the electromotive force of such a series is always given by the more rapidly wandering ion. In the lead cell it will always oppose the current, whether it be a charging or discharging one. It also always acts to lower the efficiency of the cell. The magnitude of this electromotive force may be overestimated. When the ratio of the end concentrations is at 1000 to 1, its value is only 0.09 volt. One would scarcely think this ratio could be exceeded, even in the discharging process, and we must therefore attribute the greater part of the variation in electromotive force to changes in the factors P and p.

Thermodynamical Considerations.

If a current is pressed through a storage cell or taken from the same, a change in the polarization occurs, which is determined by the variation in available energy. Upon the ionic theory, this is caused by a passage of ions from the solution to the electrode, or *vice versa*. If the temperature remains constant, the total variation in available energy is

$$\delta U - T\delta S,$$

where U represents the total energy; S, the entropy; and T, the absolute temperature.

The dependence of entropy and concentration has been treated mathematically by Planck, and his results may be applied to solutions. The electrodes or electrode surfaces may be looked upon as solid solutions containing ions.

In expanding this method of treatment we will use the following designations :[1] —

S_{mk}, C_{mk} = Entropy and concentration, respectively, of the neutral molecules in the kathode.

S_{ma}, C_{ma} = Entropy and concentration, respectively, of the neutral molecules in the anode.

S_k, C_k = Entropy and concentration, respectively, of the kation in kathode.

S_a, C_a = Entropy and concentration, respectively, of the anion in anode.

S_{wk}, C_{wk} = Entropy and concentration, respectively, of the water in kathode.

σ_m, c_m = Entropy and concentration, respectively, of the neutral molecule in solution.

σ_k, c_k = Entropy and concentration, respectively, of the kation in solution.

σ_a, c_a = Entropy and concentration, respectively, of the anion in solution.

σ_w, c_w = Entropy and concentration, respectively, of the water in solution.

n, ν = Molecule and ions, respectively.

We accordingly have for the entropy of the kathode : —

$$n_{mk}(S_{mk} - \log C_{mk}) + n_k(S_k - \log C_k) + n_{wk}(S_{wk} - \log C_{wk}),$$

and similarly, for the anode, we may write : —

$$n_{ma}(S_{ma} - \log C_{ma}) + n_a(S_a - \log C_a) + n_{wa}(S_{wa} - \log C_{wa}).$$

Finally, for the entropy of the solution, we have : —

$$\nu_m(\sigma_m - \log c_m) + \nu_k(\sigma_k - \log c_k) + \nu_a(\sigma_a - \log c_a) + \nu_w(\sigma_w - \log c_w).$$

If, under the influence of a current, a quantity, dn, of the kation passes from the solution to the kathode, we should have, as a first approximation for the change in entropy of the solution, at the kathode,

$$-(\sigma_k - \log c_k)dn,$$

[1] Dr. Jahn, in his Grundriss der Electrochemie (see this JOURNAL, Vol. IV., p. 76), makes many similar applications of Planck's method.

and for the change in entropy of the kathode,

$$(S_{mk} - \log C_{mk})dn,$$

and accordingly

$$\delta U - T(S_{mk} - \sigma_k - \log C_{mk} + \log c_k)dn$$

is the expression for the change in the kathode energy or the kathode polarization. The anode polarization is evidently

$$\delta U' - T(S_{ma} - \sigma_a - \log C_{ma} + \log c_a)dn.$$

Any increase in C_{mk} or C_{ma} increases the polarization of the cell, *i.e.* causes an increase in the available electromotive force. An increase in C_k or C_a would have the opposite effect.

For a more general case we have for the variation of entropy at the kathode

$$(S_{mk} - \log C_{mk})dn_{mk} + (S_k - \log C_k)dn_k + (S_{wk} - \log c_{wk})dn_{wk}.$$

The second term may be legitimately dropped, as the kations all pass over into neutral molecules at the electrodes when on closed circuit. The concentration of the water within the electrode may be as conveniently regarded as the concentration at the contact surface of the electrode. Neither would the argument be affected were we to speak of the concentration of the sulphuric acid instead of the concentration of the water. Both quantities are mutually dependent. We can then write the change in entropy at the kathode without much error in the following form: —

$$(S_{mk} + S_{wk} - \log C_{mk} - \log c_{wk})\,dn.$$

The corresponding change in entropy of the solution upon the kathode side is

$$-\{(\sigma_m - \log c_m)\,dv_m + (\sigma_k - \log c_k)\,dv_k + (\sigma_w - \log c_w)\,dv_w\},$$

which we may write

$$-(\sigma_m + \sigma_k + \sigma_w - \log c_m - \log c_k - \log c_w)\,dn,$$

or the entire change in entropy for kathode and solution

$$\delta U'' - T(S_{mk} + S_{wk} - \sigma_w - \sigma_k - \sigma_m - \log C - \log c_{wk} + \log c_m + \log c_k + \log c_w)\,dn.$$

This may be written in the form :—

$$\delta U'' - T\left(S_{mk} + S_{wk} - \sigma_w - \sigma_k - \sigma_m - \log\frac{C_{mk}}{c_k c_m} - \log\frac{c_{wk}}{c_w}\right)dn.$$

c_k is in all cases affected by the magnitude of c_m. When c_m is large there is relatively little change in c_k, but when c_k is small and c_w large, the tendency would be for c_k to vary rapidly under the influence of the current, and therefore cause rapid changes in the electromotive force of the cell. c_m is always small in the lead cell, and c_k must therefore be still smaller.

Instead of the product $c_k c_m$, a single factor could be written, owing to their mutual dependence. The exact analogy of the expression, $-\log\frac{C_{mk}}{c_k c_m}$, to the expression $\log\frac{P}{p}$ found in the Nernst formula, all will at once recognize. Similarly, $-\log\frac{c_{wk}}{c_w}$ represents the concentration series. But it would be more apparent if the expression for the anode polarization were written. The corresponding expression at the anode is $\log\frac{c_{wa}}{c_w}$, from which we obtain the true concentration series $\log\frac{c_{wk}}{c_{wa}}$. We have then practically the same form as the Nernst formula, and the conclusions to be drawn would therefore be the same.

To obtain the effects of change in temperature upon the electromotive force, the total thermodynamical expression for electromotive force must be differentiated as to T. This expression is so very long that it may be omitted. It reduces, however, to the v. Helmholtz equation, showing that the "heat-toning" and electrical energy are only equal at absolute zero, or when the temperature coefficient is zero.

PART II.

Experimental.[1]

In our observations we aimed to take readings for total and terminal electromotive force, and internal resistance at different times during the charging and discharging process. The cells

[1] The experimental work was done at University of Illinois, in June, 1896, with the kind assistance of Mr. H. V. Carpenter, student in electrical engineering.

were generally allowed to rest for a period equal to the time of action. From the standpoint of the theory we were particularly anxious to note the changes which took place in the cells during the first few seconds of action, or of recovery from action, both charging and discharging. We therefore had recourse to condenser readings. By means of a pendulum apparatus,[1] we were enabled to maintain constancy in the period of charge, and reduce the time of charge to about 0.1 second. With a dead-beat galvanometer we had no trouble in taking readings every five seconds when necessary. We generally alternated readings with total and terminal electromotive force. The internal resistance could then be calculated from these readings, and either a known external resistance or the known current.

We frequently had both of these quantities. In the charging process, the total electromotive force was obtained by breaking the circuit just before the pendulum reached the detents, the overturning of which charged the condenser, and then discharged it through the galvanometer. A hand break had to be resorted to for strong currents; and breaking the circuit 0.1 second before the pendulum reached its proper place, made an appreciable difference in the readings. With little practice, the circuit can be broken surprisingly near to the instant the pendulum meets the first detent. The circuit need not be broken in the charging process for terminal potential readings. This latter value should exceed the electromotive force of the cell by the fall of potential over the ohmic resistance of the cell. However, the impression is current that the electromotive force of the cell is far below the difference between the charging electromotive force and the fall of potential over the ohmic resistance.

To make our observations doubly sure, we threw the charging cell momentarily upon a discharging circuit in which the external resistance was adjusted to give us a current of about the same magnitude as the charging current. The results gave the same internal resistance in both cases, and showed in all cases the charging electromotive force to be equal to the sum of the electromotive force of the cell and fall of the potential over the ohmic

[1] PHYSICAL REVIEW, Vol. II., p. 392.

resistance of the same. Our observations were upon a 10, 20, and 40 ampere current discharge, and recovery. The charging currents were 10, 20, and 15 amperes. The latter started at 20 amperes, but we found that the machine used for charging would not give that current after the counter electromotive force of the cell reached a certain limit. We accordingly dropped to 15 amperes, which makes the curves fully as instructive. The current was regulated by a carbon and a water rheostat in series.

Tables of Results.

In the following tables, t and E denote times and electromotive force, respectively. Time is reckoned from the moment the circuit is closed or thrown open as zero. The first part of each column represents the closed circuit, and the last part of each column, the open circuit.

TABLE I.

TEN-AMPERE DISCHARGE AND RECOVERY.

I.		II.		III.	
t	E	t	E	t	E
min. sec.		min. sec.		min. sec.	
0 0	2.065	0 5	2.041	0 5	1.920
0 5	2.041	2 10	2.031	0 15	1.808
3 0	2.025	3 0	2.020	2 10	1.726
6 0	2.020	30 0	2.020	3 0	1.667
11 0	2.020	90 0	1.990	4 0	1.566
30 0	2.020	150 0	1.535	7 0	1.557
				17 0	1.530
				20 0	1.404
0 5	2.031	0 5	1.576	0 5	1.454
0 45	2.044	1 30	1.646	0 30	1.485
2 0	2.050	3 0	1.687	2 0	1.530
12 0	2.060	8 30	1.808	4 30	1.606
20 0	2.060	12 0	1.848	8 0	1.667
		52 0	1.950	25 0	1.778
				31 0	1.828
				52 0	1.898
				68 30	1.924

Table II.

TEN-AMPERE CHARGE AND RECOVERY.

I.		II.		III.	
t	*E*	*t*	*E*	*t*	*E*
min. sec.		min. sec.		min. sec.	
0 0	1.950	0 10	2.081	0 0	2.080
0 5	1.980	1 0	2.101	0 10	2.110
2 15	2.020	2 0	2.111	1 0	2.139
6 30	2.071	3 0	2.117	2 0	2.155
8 0	2.091	7 0	2.121	5 0	2.181
12 0	2.101	25 0	2.141	15 0	2.190
19 0	2.127	30 0	2.141	30 0	2.205
20 0					
0 15	2.106	0 5	2.121	0 10	2.170
1 0	2.096	1 10	2.101	1 30	2.149
3 0	2.091	4 0	2.091	3 0	2.135
4 0	2.085	30 0	2.076	10 0	2.125
9 0	2.066			30 0	2.085
20 0	2.060			990 0	2.065

Table III.

TWENTY-AMPERE DISCHARGE AND RECOVERY.

I.		III.		IV.		V.		VIII.	
t	*E*	*t*	*E*	*t*	*E*	*t*	*E*	*t*	*E*
min. sec.		min. sec.		min. sec.		min. sec.		min. sec.	
0 0	2.080	0 0	2.050	0 0	1.970	0 5	1.800	0 0	1.818
0 10	2.045	0 5	2.015	0 10	1.864	0 10	1.784	0 5	1.773
2 0	2.023	0 10	2.000	0 40	1.787	1 0	1.765	0 10	1.750
15 0	2.015	4 0	1.985	2 0	1.765	3 0	1.754	0 50	1.735
20 0	2.015	7 0	1.985	3 30	1.757	18 30	1.727	2 0	1.670
		9 30	1.875	12 0	1.740	20 0	—	4 0	1.623
		15 0	1.864	20 0	1.734			6 0	1.610
		18 0	1.777					11 0	1.584
		30 0	1.770					17 0	1.579
								19 0	1.579
								20 0	—
0 10	2.037	0 5	1.795	0 10	1.773	0 5	1.765	0 5	1.630
1 0	2.050	0 40	1.803	2 0	1.803	0 10	1.784	0 10	1.653
7 0	2.061	2 0	1.856	7 0	1.848	1 0	1.800	1 0	1.697
12 0	2.061	4 0	1.925	10 0	1.856	3 0	1.811	5 0	1.742
20 0	2.068	9 0	1.962	13 0	1.864	6 0	1.818	13 0	1.788
	2.068	20 0	1.970	20 0	1.879	15 30	1.826	20 0	1.788
						20 0	1.833	1020 0	2.004

TABLE IV.

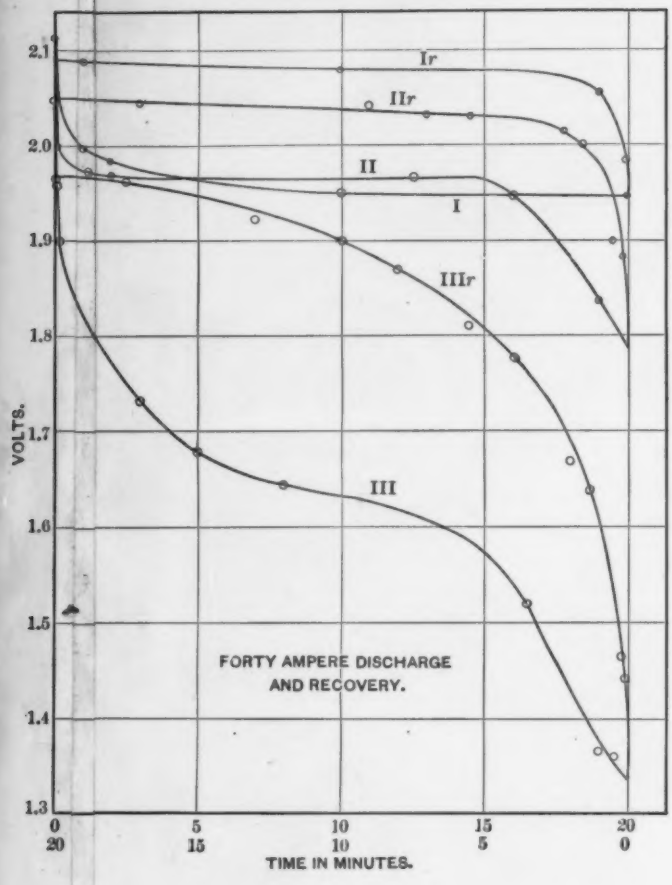
TWENTY AND FIFTEEN AMPERE CHARGE AND RECOVERY.

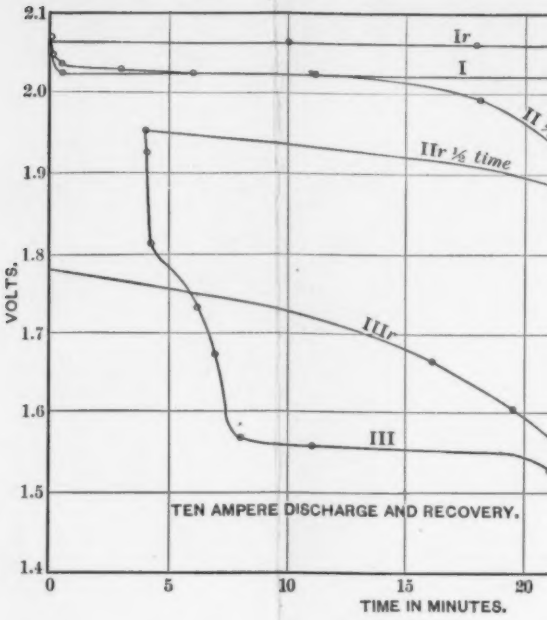
I.		II.		III.		IV.	
t	*E*	*t*	*E*	*t*	*E*	*t*	*E*
min. sec.		min. sec.		min. sec.		min. sec.	
0 0	2.004	0 25	2.125	0 0	2.060	0 0	2.098
0 5	2.037	8 30	2.197	0 3	2.091	0 5	2.184
0 10	2.067	18 0	2.205	0 15	2.174	0 20	2.257
1 0	2.106	20 0	2.219	2 0	2.287	2 0	2.371
1 30	2.114			8 0	2.394	6 0	2.490
3 30	2.151			13 0	2.424	11 0	2.492
5 0	2.166			14 30	2.432	14 30	2.500
12 0	2.197			20 0	2.440	20 0	2.500
19 50	2.201						
20 0	—						
0 5	2.166	0 5	2.174	0 10	2.311	0 5	2.424
0 40	2.106	0 30	2.122	0 20	2.212	0 10	2.364
1 20	2.098	1 0	2.106	1 0	2.174	1 0	2.235
2 0	2.087	2 0	2.091	6 0	2.136	4 0	2.182
10 0	2.057	14 0	2.045	7 30	2.129	6 30	2.166
19 0	2.045	20 0	2.045	19 0	2.091	20 0	2.130
20 0	—			20 0	—		

TABLE V.

FORTY-AMPERE DISCHARGE AND RECOVERY.

I.		II.	
t	*E*	*t*	*E*
min. sec.		min. sec.	
0 0	2.115	0 5	2.000
0 5	2.050	1 10	1.971
1 0	1.998	2 0	1.970
2 0	1.985	12 30	1.965
10 0	1.950	16 0	1.948
20 0	1.946	19 0	1.838
		20 0	—
0 5	1.985	0 5	1.880
1 0	2.055	0 30	1.900
10 0	2.081	1 30	2.000
19 0	2.090	2 10	2.017
20 0	2.091	5 30	2.029
		7 0	2.032
		9 0	2.041
		17 0	2.047
		20 0	2.050


Ir
IIr
II
I
IIIr
III
FORTY AMPERE DISCHARGE
AND RECOVERY.
VOLTS.
TIME IN MINUTES.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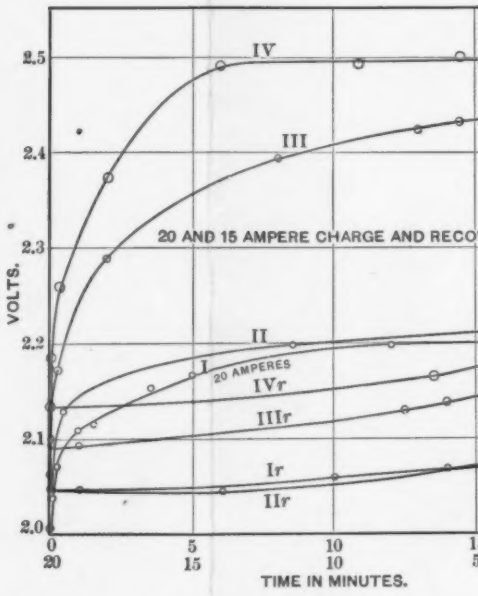
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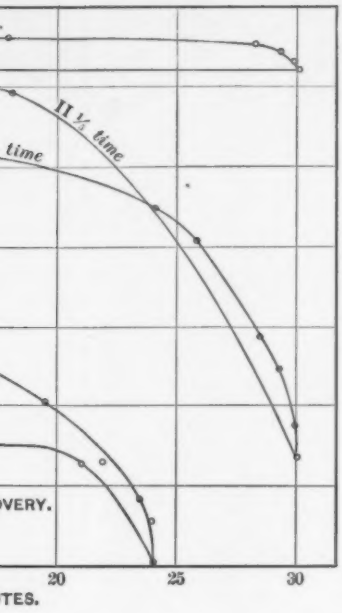

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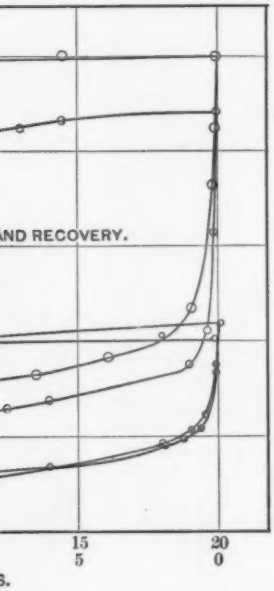

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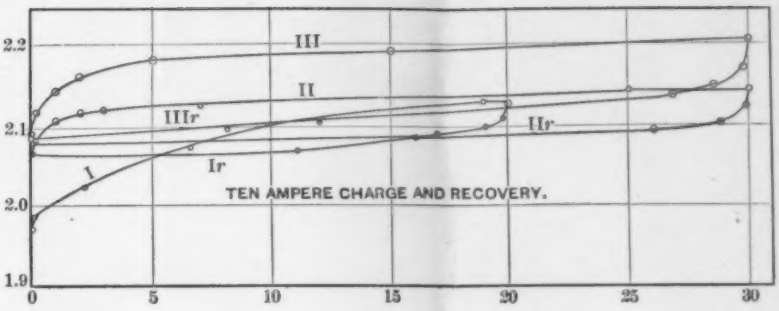
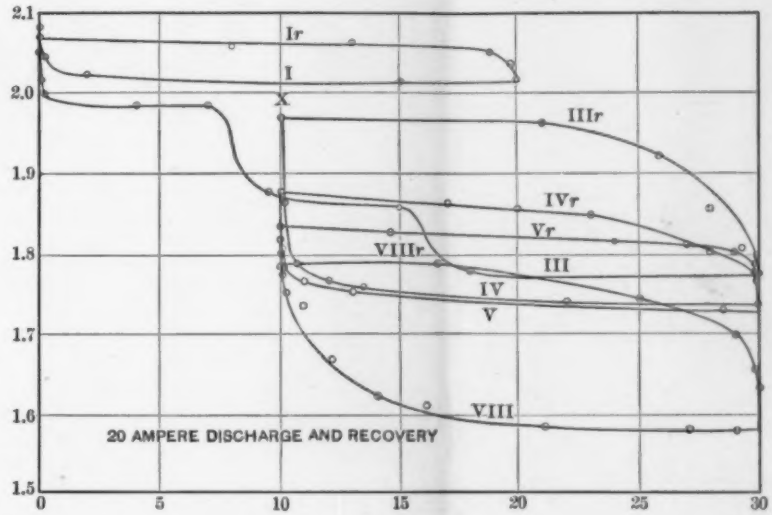
LEAD CELL.





Curves.

In all curves, the recovery was plotted in the inverse order. This enables one to compare with greater ease the successive runs. The discharges are marked in the order of their occurrence; and the corresponding recovery curves are similarly marked, but with the subscript r added. In the 10-ampere charge between II. and III., is a period of charge equal to forty ampere hours. In the 20-ampere discharge, II. is nearly the duplicate of I., and neither data nor curves are inserted; and VI. and VII. were omitted, to leave the curves less cumbersome. The value which VIII. obtained after 1020 minutes is indicated on the curve by an x. In the curves marked 20 and 15 ampere charge, the first only is a 20-ampere charge; and during the last five minutes, it sank below 19 amperes. Between II. and III., there is a period of charge equal to seventy ampere hours, after which the cells rested one hour and forty-five minutes, before the charging was again begun. Between III. and IV. was a rest over night, followed by twenty-five ampere hours' charge, and then a period of two hours' recovery. The observations on 40-ampere discharge were continuous.

The curves in general show great regularity, and their character is in harmony with the theory. The drop in electromotive force on closing the discharging, or the rise in electromotive force on closing the charging, current does not by any means take place instantaneously. In passing from charging to discharging, the curve of electromotive force would not be discontinuous. Let us notice the 20-ampere discharge and recovery curves. The cells, to begin with, were not charged to their full capacity, but the electromotive force in zero time, curve I., is about the normal value. In one minute, a marked change has occurred. It would not take a great stretch of the imagination to see that, if observations were frequent and delicate enough, the character of this curve could be logarithmic, as demanded by the Nernst formula in establishing a concentration series, and in producing new value of p, the osmotic pressure of the ions, brought about by bringing lead sulphate into solution more rapidly than it is precipitated from the same. The new conditions arrive at equilibrium in about one minute, and the change in electromotive force is about

constant for the remaining nineteen minutes. Upon throwing the circuit open, the reverse action occurs. The concentrated acid diffuses and changes in ionic pressure, also return to something near the original value. Were the series purely a concentration series, the thickness of the denser strata could be calculated when the time of diffusion is one minute. The strata of the more dilute acid would then not exceed 0.1 mm., and assuming ionic changes, the thickness would be less. The point to which I_r recovers at the end of twenty minutes shows approximately the change in electromotive force due to change in P, or electrode substance, and to change in p, due to increased bivalent $\overset{++}{Pb}$ ions. The influence of the lead sulphate upon the plate until after fifteen ampere hours have been discharged is small. We think the character of curve III. is probably more uniform than noted, and that the error arose from inaccurate adjustment of the current. The thickness of a layer of lead required for fifteen ampere hours' discharge in this cell is 0.01 mm. The sulphate deposited would naturally be thicker, but that is more than doubly compensated by the porosity of the plate. Whatever the thickness or the thinness of the sulphate, it apparently begins, in this case, to exert its influence upon the solution pressure of the peroxide after about fifteen ampere hours have been discharged; and as the deposit grows, to leave a more marked effect upon the later discharges. More or less diffusion of peroxide through the soft and thin layer of lead sulphate might take place. The deposit need not be perfectly uniform. It shows great porosity, is filled with dilute acid, all combining to aid the peroxide in coming in contact with the solution. It is a well-known fact that mixed electrodes act as electrodes of a single metal as long as that metal is present in considerable mass and has a larger solution pressure than the other elements. Such, for example, is the zinc-mercury amalgam electrode used in the Clarke cell. We are in all cases concerned only with the electrode surface, not with the interior of the electrode. A strong current taken from a Clarke cell removes the surface zinc more rapidly than it can be supplied from within to restore equilibrium. But upon resting, the electromotive force

regains its value. This process may be continued until a large part of the zinc is exhausted, and the cells still show the solution pressure of the zinc. The cell, aided by the zinc sulphate crystals, shows no change in p. Were these crystals to harden over the amalgam and prevent liquid contact, little could be expected of the cell. This is somewhat analogous to the lead cell.

The lead sulphate is incrusted upon the plate, is very thin, and is generally found well saturated with the peroxide. If the electromotive force at any instant is given by the PbO_2 in contact with the solution, and the number of ions in the solution, the drop would be greater when a coating begins to form over the peroxide. The current would use up the peroxide more rapidly than the fresh peroxide is able to come in contact with the solution. This effect would be magnified as the coating grew. During rest, the peroxide would still continue to diffuse as in the zinc amalgam electrode, the solution pressure would continue to rise, and when the cell is well discharged, would take hours to reach near the original value. The x on the curve sheet under discussion shows the value to which VIII, has attained after seventeen hours.

The presence of the sulphate is also attested by the increased internal resistance, though its effect is much less than what would be expected of pure sulphate. But it must be remembered how thin the stratum of sulphate is, and that it is intermixed with peroxide, soft and thoroughly wet with dilute acid. Gladstone and Hibbard maintain that the pores are partly shut up by the sulphate during the last stages of the discharge, and the gases formed cannot escape, which causes an increase in internal resistance. To this, it must be said that gases are not occluded at an electric pressure as high as that required for evolution of the same. According to the second law of thermodynamics, gases could no more pass from a lower to a higher pressure of themselves than heat could pass of itself from a lower to a higher temperature. Occluded gases could not, therefore, reach the gaseous state during discharge, but must pass directly from the occluded to the liquid condition at the same electromotive force at which they were formed. No doubt H and O are, to a slight extent, present, and would behave as here noted, the process

being a reversible one. The formation of a free gas, however, is irreversible, and belongs to a much higher electrical pressure. With black platinum electrodes, the former occurs at 1.07 volts, and the latter at 1.70 volts. What their respective magnitudes are in the lead cell cannot be definitely stated; but, for reason mentioned later, the irreversible evolution of H occurs at a much higher value, and it is probable that the occlusion point is also higher. The following consideration also refutes the claim of Gladstone and Hibbard. If the gases were the cause of increased resistance during discharge, the slow escape and reabsorption of the same during a period of rest would produce a diminution in the resistance of well-discharged cells. The opposite effect we have found to take place. The internal resistance actually rises and becomes nearly double after long periods of rest, in spite of the diffusion of the acid at the electrodes, which would lower slightly the resistance. This latter effect may be too small to be appreciable. This explanation offers itself to the writer: the sulphate, on standing, becomes more compact, and therefore would have very little acid distributed through it.

Ayrton has shown that the rate of consumption of the PbO_2 upon the plate is uniform. Gladstone and Hibbard, basing their conclusion upon Ayrton's work, maintain that it is shown conclusively that the fall in electromotive force during discharge cannot be due to the amount of PbO_2, our P_+ factor, upon the plate. With this conclusion the writer cannot agree. A constant decrease in PbO_2 at the electrode surface would mean a logarithmic decrease in the electromotive force of the cell, did not other factors before mentioned enter into consideration. A cell discharged continuously at constant current shows that the curve, debarring the initial drop, is more nearly a logarithmic one than any other through a wide range. The question, however, is not how much PbO_2 one may determine to be upon the plate, but how much is in actual contact with the solution. Gladstone and Hibbard carried on some valuable experiments upon the effects of the concentration of the acid upon the electromotive force of the cell. The concentration of the acid certainly must modify the *factors* which are *effective* in producing an electro-

motive force, and in so far the electromotive force may be said to depend upon the concentration of the acid. But to regard the acid as the cause of the electromotive force is a step too far. Wade, in his criticism of Gladstone and Hibbard's work, — and Barnett endorses his objection, — says concerning their theory, "While extremely plausible, it is hardly consistent with the whole facts of the case. If it were correct, the local weakening of the acid on discharge ought to be equally marked, provided the current in both cases is the same, and the accompanying electromotive force as much too low as the other is too high. Instead of this, it will be always found that, throughout the greater part of the discharge, the electromotive force is but slightly below what is required to correspond to the strength of the acid indicated by a hydrometer inserted in the main body of the liquid." This objection seems well founded from the standpoint of Gladstone and Hibbard, and a hydrometer may, when the cell is at rest and readings are taken with a condenser, indicate the electromotive force, but when the cell is in action it cannot do so. The question is, "whether the accompanying electromotive force is as much too low, upon discharge, as the other is too high." The charging electromotive force Gladstone and Hibbard recognize as being too high. Our discharge curve shows that inside one minute, values are reached far too low, and simple inspection of the curves for the charging process shows that the behavior of the lead cell during the charging process is analogous to the behavior during discharge. The various factors change in the inverse order to that already noted, and need not be further discussed.

The claim is frequently made that by-products such as H_2O_2 and $H_2(SO_4)_2$ are formed during the charging process. Such formations may be quite possible, but why regard them as by-products? When the cell is well charged, and the charging process still continues, the sulphate is all, or nearly all, used up, the acid at the electrode surface is nearly pure, and the electromotive force necessary to overcome the electromotive force of the cell must be very large. The products of ionization at the electrodes are $\overset{++}{Pb}$ and $\overset{--}{SO_4}$, $\overset{++}{H_2}$ and $\overset{--}{SO_4}$, and $\overset{+}{H}$ and $\overset{-}{OH}$. The lead ions present are no longer supplied sufficiently rapidly to carry

the current; the $\overset{+}{H}$ ions corresponding to the $\overset{-}{OH}$ are always small; and the rapid removal of the principal positive ion, viz.: $\overset{+}{H}$, means a supersaturation of $\overset{=}{SO_4}$ and $\overset{-}{OH}$ ions at the electrodes. This condition cannot be maintained, and it is avoided if we assume the immediate and direct formation of $SO_4 - H - H - SO_4$ and of $OH - HO$. As a forced condition, the electromotive force necessary for the formation would be above the ordinary. The electrode products are liberated, and the process becomes irreversible. The gases so liberated are not formed at the same electromotive force at which gases are occluded. The latter is a reversible process, the former generally is not, and cannot be, under conditions existing in the ordinary lead cell. The energy consumed is largely lost. It only succeeds in tearing up the plate, *i.e.* in making it more porous, and in bringing a little more lead in contact with the liquid. This, of course, slightly increases the output of the cell. That such products as $H_2(SO_4)_2$ and H_2O_2 should break up under rapid evolution of heat seems quite plausible. However, it does not explain the heating at the electrodes, for we know that the products must have been formed under absorption of an equivalent amount of heat energy; and if, therefore, no such product had been formed at all, the amount of heat generated would have been exactly the same as if all of the products had broken up, instead of a small quantity remaining in the solution. Either case would lead us back to the v. Helmholtz equation for an explanation of the heat developed. We must accordingly assume a large divergence in the value of the "heat-toning" and of the electrical energy at this electromotive force.

The data and curves for terminal potential are not given. The internal resistance for curve I., 20-ampere discharge and recovery, was 0.006 ohm. At the end of one minute this value fell to 0.0048 ohm, which value it retained until curve III. is reached, when it again increases and reaches at the end of run VIII. 0.0072 ohm. After standing seventeen hours the resistance of this cell had increased to 0.01 ohm. A few readings were then taken at intervals of five to ten seconds, while we were adjusting the external resistance to give a 10-ampere current. The sixth reading showed an internal resistance of 0.0065 ohm. The intermediate readings

show no great regularity, as neither time nor current was observed with accuracy. Periods of rest also existed between the few seconds' run. We had taken the readings chiefly to see if the galvanometer was working rightly, and the readings were accidentally preserved, and what had occurred dawned upon the writer after he had no further opportunity to repeat the observations. In every case, however, where the period of rest from discharge is longer than common, the internal resistance increases.

Now, if a small current, say of one or perhaps even five amperes, was taken from the cell after a period of rest, the drop in total electromotive force would be small for a short time, and the terminal potential readings would go up, owing to decreased internal resistance. This is perhaps what was really noted by those who state that the electromotive force rises when a current is first taken from a cell.

The internal resistance was generally smaller for strong currents, but the difference may be within the limits of experimental error. The lowest internal resistance noted was 0.0032 ohm, and was observed at the maximum of curve II., 15-ampere charge. This may probably be due to the increase in the concentration of the acid formed at the electrode and diminution of lead sulphate. At the maximum of curve IV. the terminal potential indicated an internal resistance of 0.0081 ohm. This increase in resistance may safely be attributed to liberated gases, and to the *very* concentrated acid. These wide variations need farther investigation.

Every curve incloses an area representing so much lost work, which, in the very nature of the cell, it is impossible to obviate. Both theory and observation show that these areas will be smaller with small currents, other conditions remaining the same. Curves IV. and IV_r, 15-ampere charge, embrace an area three times as large as the C^2R loss within the cell. This area, figured out in heat units, represents about 23 gram-calories, which would not cause a very appreciable rise in temperature, as the cell consisted of 92.4 Kg. of lead, and 61.6 Kg. of solution and glass. The other curves more generally show even smaller magnitude, and indicate the difficulty there would be encountered in trying to determine this loss in heat units. We are further at a loss to

know how many heat units to expect; for the cell can work at only *one* electromotive force, according to the v. Helmholtz equation, at which there will be no absorption nor abstraction of heat. This electromotive force is the normal electromotive force of the cell if, as before mentioned, the temperature coefficient is zero. Of this we are not certain, and we shall therefore designate it by E_0. The normal electromotive force of the cells in question certainly lies somewhere between 2.0 volts and 2.1 volts. To charge above E_0 or discharge below it would, according to the equation, produce heat. The greater the divergence of the electromotive force from E_0, the greater the quantity of heat absorbed or generated. It is probably much easier to hold the charging electromotive force far above this value for a considerable period than to hold the discharging current below it, and hence the heating effects would be more easily noted in the former process. This difference could be made more apparent if E_0 were below the normal electromotive force. We could then have cooling during certain stages of the discharge of a cell. The inclosed area could only represent developed heat, if all or the great part lies below the horizontal line for E_0. This leaves the explanation of the disappearance of the lost energy in practically the same condition in which the author found it. Numerous attempts have been made to get some light upon the problem by determination of the temperature coefficient at widely different electromotive forces. All efforts were baffled, and it seems almost impossible to maintain sufficient constancy in the cell for such a determination.

It is commonly observed that much energy is wasted in discharging the cell below about 1.7 volts. It is equally noticeable that there is a corresponding loss when the charging is carried above 2.25 volts. Practically it is questionable whether one should charge at all with a 15-ampere current, unless time be a grave consideration.

When large currents, like 40 amperes or more, are taken from a cell, it is quite impossible to get a high efficiency, owing to the establishing of a concentration series and the large changes which p and P undergo. These are inherent difficulties too little recognized in the electric storage car problem.

UNIVERSITY OF NEBRASKA, November, 1896.

THE INFLUENCE OF A STATIC CHARGE OF ELECTRICITY UPON THE SURFACE TENSION OF WATER.

By Edward L. Nichols and John Anson Clark.

THE influence of a static charge upon the form and behavior of a water jet is well known. The phenomena formed a favorite subject among the earlier electricians.[1] It has commonly been ascribed to a change in the surface tension of the liquid. J. W. Draper described some experiments showing the influence of electrification upon capillary phenomena,[2] in 1845. In 1861 Tate proposed to use the rate of discharge of a siphon to measure the degree of electrification of the vessel with which it was connected.[3] In 1870 Van der Mensbrügghe published an account of his interesting experiments upon electrified liquid films, in verification of his views concerning surface tension.[4] So far as the present writers know, however, no direct measurements touching this point have been published.

Fig. 1.

It is the object of the investigation described in this paper to determine quantitatively the influence of an electric charge upon the surface tension of water. The method of determining surface tension consisted in weighing drops. The water dropper, the general construction of which is shown in Fig. 1, consisted of a burette of 100 c.c. capacity, which served as a reservoir for the liquid. This was provided with an orifice made from a piece of capillary tubing about 6 cm. long. One end of this was melted,

[1] Nollet, Recherches sur les causes des phénomènes électriques (1749), p. 327; Beccaria, Dell' ellettricismo artificiale (1772); Carmoy, Journal de physique (1788); Singer, Elements of Electricity and of Electrochemistry (1814); Faraday, Experimental Researches (Art. 1571).

[2] Philosophical Magazine (3), Vol. 26, p. 185.

[3] Philosophical Magazine (4), Vol. 21, p. 452.

[4] Poggendorff's Annalen, Vol. 141, p. 287.

and was then "upset" until its diameter was increased to about 1 cm. The upset portion was then ground off so as to form a plane surface at right angles to the axis of the tube, and this surface was ground at the edges until a circular disk was obtained. The water after passing through the capillary tube floated out upon the ground-glass surface, where it remained until a sufficient amount had gathered to form a drop. Figure 2 shows this orifice with the drop clinging to it. With this arrangement, which was the result of many trials with other forms of water dropper, the drops were found to be very nearly of the same weight.

Fig. 2.

In order to determine the degree of accuracy with which surface tension could be measured with this apparatus, 16 series, consisting of 20 drops each, were collected and weighed successively. The results are given in Table I.

TABLE I.

DATA FOR THE CALIBRATION OF THE DROPPER.

Number of drops.	Weight of beaker and water.	Weight of beaker.	Weight of the water.	Deviation from the mean.
20	28.5951	25.8230	2.7721	+0.0015
20	28.5935	25.8230	2.7705	−0.0001
20	28.5942	25.8230	2.7712	+0.0006
20	28.5906	25.8230	2.7676	−0.0030
20	28.5924	25.8230	2.7694	−0.0012
20	28.5961	25.8230	2.7731	+0.0025
20	28.5912	25.8230	2.7782	+0.0076
20	28.5944	25.8230	2.7714	+0.0008
20	28.5943	25.8230	2.7713	+0.0007
20	28.5926	25.8230	2.7696	−0.0010
20	28.5933	25.8230	2.7703	−0.0003
20	28.5864	25.8230	2.7574	−0.0132
20	28.5935	25.8230	2.7705	−0.0001
20	28.5912	25.8230	2.7782	+0.0076
20	28.5932	25.8230	2.7702	−0.0004
20	28.5925	25.8230	2.7695	−0.0011
Mean			2.7706 g.	0.0026

It will be seen that the mean value of the weight of 20 drops is 2.7706 g., and that the average deviation from the mean is 0.0026 g., which is less than the tenth of one per cent.

Since the surface tension of water is a function of the temperature, it was necessary to reduce all weighings to the same temperature, for which purpose the temperature of each set of drops had to be determined. For this purpose a thermo-couple, consisting of a piece of No. 20 German silver wire and of copper wire of the same size, was used. One junction was placed in a glass tube about 12 cm. long and 1 cm. in diameter, which was closed at one end. The junction within the tube was surrounded with molten sulphur, which was allowed to cool. This tube was then suspended by an insulating rod in a glass beaker, which was also insulated. The beaker was filled with distilled water, which was kept boiling slowly by means of a Bunsen burner. The other junction was placed in such a position that just as each drop left the orifice, it completely surrounded the junction. In this way the temperatures of the drops were measured at the instant when they were detached from the orifice, a method which was found to give a much better result than any attempt to measure the temperature of the liquid after it had been collected for weighing.

The wires for this thermo-couple were supported by insulating glass rods, which carried them to a sensitive galvanometer with high resistance coils. The galvanometer itself was likewise well insulated. It was necessary to insulate this entire circuit, including the instrument, because the thermo-element was brought into contact with the charged water at the instant of dropping. The galvanometer needle was so heavy that the static charge had no appreciable effect upon its movements. By means of this arrangement, it was possible to read the temperature of the drops accurately to tenths of a degree. In order to open and close the electric circuit without discharging the system of which it formed a part, a porcelain cup of mercury was placed upon an insulating stand, and one terminal of the galvanometer was inserted permanently into the mercury. The other terminal was fastened to a copper disk furnished with a vulcanite rod 25 cm. long which served as a handle. This disk was allowed to float upon the surface of

the mercury. By raising it the circuit was broken, and the swing of the galvanometer needle was observed.

The thermo-element was calibrated in the usual manner by placing the variable junction in a water bath, the temperature of which was determined by means of a thermometer reading to tenths of a degree. The successive readings were plotted, and a curve of calibration was obtained.

By means of the arrangement just described, the weight of a drop of water was determined as a function of the temperature

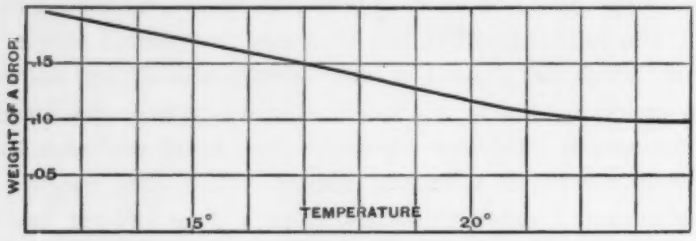


Fig. 3.

for the range of temperatures through which it was necessary to work. The data for this temperature correction are given in Table II., and the results are shown graphically in Fig. 3.

TABLE II.

DATA FOR THE TEMPERATURE CORRECTION CURVE OF THE DROPPER.

Galvanometer throw.	Temperature.	Weight of 20 drops.	Weight of 1 drop.
31.1	17.8	2.801	0.140
31.0	18.0	2.782	0.139
29.0	22.3	2.001	0.100
28.6	23.1	1.982	0.099
28.2	24.0	1.963	0.098
28.2	24.0	1.962	0.098
28.3	23.7	1.970	0.098
29.2	22.0	2.103	0.105
29.7	20.8	2.311	0.115
30.3	19.4	2.632	0.131
30.8	18.3	2.705	0.135
31.4	17.1	3.012	0.156
32.3	15.2	3.281	0.164
33.6	12.4	3.905	0.195

In order to obtain a more constant potential than could be secured with the Holtz machine already mentioned, a battery of 24 Leyden jars was placed in the static circuit. By the use of these jars, and by carefully regulating the speed of the motor, it was possible to obtain a fairly constant potential for considerable intervals of time. The potential was varied by varying the speed of the motor. The Holtz machine was placed in an adjoining room in order to avoid, as far as possible, disturbances due to induced charges.

For the measurement of the potential, an electrometer was devised in which the repellent forces between a charged ball and a vertical plate were balanced against the force of gravity. This principle, which is very old, having been employed by Henley in his quadrant electroscope in the last century, was applied in such a manner as to make it possible to compute potential in absolute measure with fair approximation. In its essential features, this instrument consisted of a metal ball suspended by means of a long wire, in such a manner that its center coincided with the axis of a vertical disk of metal. To obtain a suitable ball for the range of potential which it was desired to measure, large leaden shot were heavily plated with copper. The lead was then removed by fusion in the luminous flame of a Bunsen burner, leaving behind a hollow sphere of copper. This sphere was reduced in weight by further suspension in an electrolytic bath, in which the sphere was the anode. In this way several light and fairly accurate metal spheres were obtained. The suspension wire was of German silver No. 41, which had been straightened by heating it to a dull red under tension.

The dimensions of the electrometer were as follows:

Length of the suspension wire	73.750 cm.
Radius of disk	15.750 cm.
Mass of repelled ball	0.142 g.
Radius of the ball	0.436 cm.
The value of the force due to gravity was taken as	980.380 dynes.

The method of using this instrument consisted in connecting the plate and the ball, the latter by means of its suspension wire, with the charged system of which the water dropper forms a part.

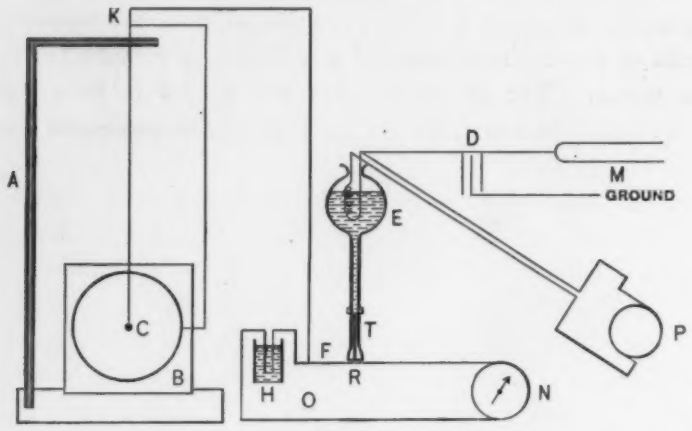


Fig. 4.

Figure 4 is a diagram showing the electric connection between the machine, the battery of Leyden jars, the electrometer, and the water dropper. In that diagram

A is the glass insulating rod;
B is the glass plate;
C is the metallic disk with ball at its center;
D is the condenser;
E is the German silver coil for heating water;
F is the point of connection of the electrometer;
G is the ground circuit;
H is the vessel of constant temperature;
K is the point of support of the electrometer ball;
M is the terminal of the Holtz machine;
N is the galvanometer;
O is the constant junction of the thermo-couple;
P is the dynamo;
R is the dropping orifice with the variable junction of the thermo-couple beneath it;
T is the reservoir and stem.

When charged, the sphere is repelled from the disk and assumes a position of equilibrium in which electric repulsion is balanced by the repellent force due to gravity. In the computation of the potential as measured by the deflection of the ball, the change of surface tension at the edges of the disk was neglected. The method of computation was as follows:

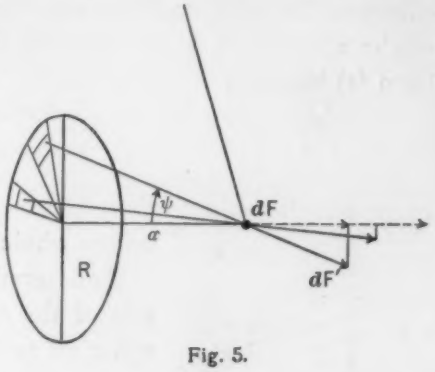


Fig. 5.

In Fig. 5 let

R = radius of the disk;
ρ = radius of the sphere;
a = distance of the sphere from the plate;
σ = surface density of charge;
r = variable distance of an element of area from the center of the disk;
c = variable distance of an element of area from the sphere;
L = length of the suspension wire;
$rd\theta \cdot dr$ = element of area;
M = mass of the sphere;
ϕ = the angle of deflection;
V = the potential;
q = the charge on the sphere;
ψ = angle between a and 1;
dF' = force due to charge on element of area;
dF = horizontal component of force;
F = force required.

We have then:

$$dF' = q\frac{d^2}{c^2} = \frac{q \cdot \sigma r d\theta dr}{c^2};$$

and

$$dF = dF' \cos\psi = \frac{q\sigma \cdot rd\theta \cdot dr}{c^2} \cdot \cos\psi. \qquad (1)$$

Now the quantity $rd\theta \cdot dr \cdot \cos\psi$ is the element of area projected on a plane normal to the line drawn from the sphere to

the element; and this projected area, when small, divided by the square of the distance between the sphere and the element is, by definition, the solid angle at the center of the sphere subtended by the element of area. Let the solid angle be denoted by ω. Then (1) becomes

$$dF = q\sigma d\omega. \tag{2}$$

$$\therefore\ F = q\sigma\omega, \tag{3}$$

where ω is the solid angle at the center of the sphere subtended by the whole disk.

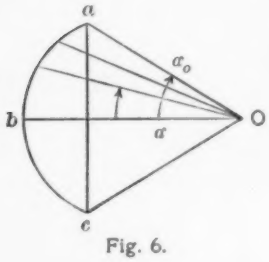


Fig. 6.

To determine ω, let ac (Fig. 6) be a section of the disk and abc a section of the spherical surface with O as a center. Take as element of area an annulus on the spherical surface equal to $2\pi r \sin\sigma \cdot rd\sigma$

$$d\omega = 2\sin\sigma d\sigma;$$
$$\omega = 2\pi \sin d.$$

The capacity of a sphere is equal to its radius:

$$\therefore\ C = \frac{q}{V} = \rho \text{ and } q = \rho V.$$

The capacity of a circular plate is $\frac{2R}{\pi}$:

$$\therefore\ C' = \frac{I}{V} = \frac{\pi R^2\sigma}{V} = \frac{2R}{\pi},$$

and

$$\sigma = \frac{2V}{\pi^2 R}.$$

The return force due to gravity is $Mg\sin\phi$, or $Mg\frac{a}{L}$.
Making this substitution in (3), we have

$$mg\frac{a}{L} = \rho V \cdot \frac{2V}{\pi^2 R} \cdot 2\pi\left[1 - \frac{a}{\sqrt{a^2+R^2}}\right];$$

$$V^2 = mg\frac{a}{L} \cdot \frac{\pi R}{4\rho} \Big/ 1 - \frac{a}{\sqrt{a^2+R^2}} = \frac{\pi mgaR\sqrt{a^2+R^2}}{4\rho L[\sqrt{a^2+R^2}-a]}$$

$$= K\frac{a\sqrt{a^2+R^2}}{[\sqrt{a^2+R^2}-a]}.$$

In this instrument $L =$ 73.7500 cm.;
$R =$ 15.7500 cm.;
$M =$ 0.1420 grams;
$\rho =$ 0.4360 cm.;
$g =$ 980.3800 dynes;
$V^2 =$ 53.5540.

The method of reading the deflection of the electrometer was as follows: A telescope, within the eyepiece of which a ruling on glass consisting of fifty equidistant lines had been mounted, was set up at a distance of about 500 cm. The telescope was adjusted so that the zero line of the scale coincided with the plane of the disk of the electrometer. The field of the telescope was lighted by two large fish-tail burners placed behind the electrometer, so that the fine wire, by means of which the ball was suspended, was seen projected upon a bright background. The definition was such that it was easily possible to estimate tenths of a scale division. A calibration of the eyepiece by means of a standard scale showed each division to be equivalent to 0.2127 cm. The telescope for reading the galvanometer and that for reading the electrometer were so placed with reference to each other as to enable the observer to make alternate readings by merely turning his head.

When the method of experimenting above described was put into operation, a number of interesting phenomena were observed. The first noticeable effect of charging the water was the change in the rate of dropping. Even with a very small charge this increase was easily noticeable. As the charge was increased, the size and shape of the drop could be seen to vary. The accumulated water on the disk of the dropper gradually became cone-shaped; and when the charge was further increased, this cone-shaped body of water broke up into three and sometimes four similar cones, from the apex of each of which issued a fine stream of water. These little cones stood out from each other like so many similarly charged gold leaves. While they generally retained the same relative positions to one another, they did not always remain the same with reference to any particular point on the disk. At times they were all found to be revolving around the central orifice of

the disk, while at others they had a harmonic motion of slow period.

The mode of procedure in making measurements was as follows:

(1) The temperature was read.
(2) The voltage was read.
(3) A series of drops was collected.
(4) The voltage was read again.
(5) The temperature was read again.
(6) The water was weighed.

The voltage was then changed, and readings were taken again in the same manner. Readings were taken nearly every day for three months. The data obtained were collated, and the results were used in plotting a curve, with the relation between the weight of a single drop as ordinate, and the deflection of the repelled ball of the electrometer as abscissæ. The data from which this curve was plotted are given in Table III.

TABLE III.

Scale divisions of electrometer readings.	Weight of 1 drop corrected to 20 degrees, in grams.	Scale divisions of electrometer readings.	Weight of 1 drop corrected to 20 degrees, in grams.
0	0.120	35	0.113
20	0.119	36	0.111
21	0.119	37	0.109
22	0.118	38	0.106
23	0.118	39	0.104
24	0.117	40	0.102
25	0.117	41	0.100
26	0.116	42	0.099
27	0.116	43	0.097
28	0.116	44	0.095
29	0.115	45	0.093
30	0.115	46	0.090
31	0.115	47	0.088
32	0.115	48	0.085
33	0.114	49	0.080
34	0.114	50	0.075

Since it was the object of this investigation to determine the *variation* of surface tension with electrification, no attempt to

measure the surface tension itself was made. Numerous such measurements exist, and the average of the following values taken from a table published in *The Philosophical Magazine* has been

TABLE IV.

TABLE OF VALUES OF SURFACE TENSION.

Tension at 0°.	Observer.	Tension at 0°.	Observer.
77.4	Artur.	77.1	Jaeger.
75.2	Brunner.	73.7	Magie.
75.2	Desain.	73.3	Quincke.
75.0	Frankenheim.	74.3	Rodenback.
75.4	Frankenheim.	79.7	Sondhauss.
75.6	Frankenheim and Sondhauss.	73.6	Volkmann.
75.4	Gay-Lussac.	75.8	Volkmann.
75.2	Gay-Lussac.	76.5	Wolf.
74.5	Hagen.	77.3	Wolf.

Average, 75.4.

used as a reference value. The values given in the table are for 0°. By applying Brunner's formula[1] for the variation of surface tension with temperature to the mean of the values given in the table, we

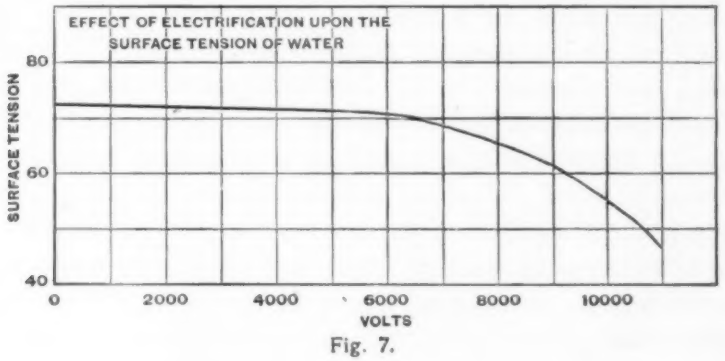


Fig. 7.

obtain 72.5 as the value of the surface tension of water at the temperature of 20°. This is the temperature to which our readings were all reduced by means of the curve in Fig. 3. By making use of the readings for voltage as deduced from the deflection of the

[1] Brunner, Poggendorff's Annalen, 70, p. 481.

electrometer, it is possible to plot a curve which shows the relation between the surface tension and the voltage to which the water was subjected. From this curve, which is given in Fig. 7, and from Table V., it will be seen that the surface tension diminishes

TABLE V.

SURFACE TENSION AND ELECTRIFICATION.

Volts.	Surface tension.	Volts.	Surface tension.
0	72.50	8,000	66.75
2000	72.00	10,000	54.50
4000	71.80	11,000	46.60
6000	71.25		

slowly, but at an increasing rate with rise of potential, and that at 11,000 volts it has fallen off from 72.5 to about 46.6 in value.

Having reached this result, it was thought of interest to determine whether the change of surface tension with electrification was dependent upon the sign of the charge. For this purpose, one terminal of the Holtz machine was grounded, and the other terminal, which was found by testing with the proof plane to be positive, was attached to the apparatus. After charging, the machine was stopped, and a series of drops were gathered and weighed. The temperature was noted, and the weighings collated as described above. This process was repeated 11 times. The terminals of the machine were then reversed, and the voltage was brought to the same point as before, and the measurements were repeated with the same range of potential as before. The results obtained with positive and negative charges are given in Table VI.

TABLE VI.

DATA.

Weight of one drop corrected to 20 degrees. Average voltage 7000.

Positive.	Negative.	Positive.	Negative.
0.115	0.114	0.115	0.115
0.115	0.115	0.115	0.115
0.115	0.115	0.115	0.115
0.114	0.115	0.114	0.115
0.115	0.115	0.115	0.115
0.115	0.114		

It will be seen that there is no appreciable difference in the effects of the positive and of the negative charge.

From the foregoing experiments, the following conclusions may be drawn:

(1) The surface tension of water is decreased by giving it a static charge of electricity.

(2) The law of decrease is independent of the nature of the charge. Although no quantitive measurements were made upon other liquids, some preliminary determinations lead the writers to believe that a charge of static electricity exerts a like influence upon the surface tension of other liquids.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1896.

ON THE MECHANICAL CONCEPTIONS OF ELECTRICITY AND MAGNETISM.[1]

BY W. S. FRANKLIN.

THE following development is based upon the conceptions of Maxwell. It follows, in some particulars, the work of Lodge. The conceptions here developed, of the dependence of electromotive force upon changing magnetic flux, of the dependence of magnetomotive force upon changing electric flux, of the production of electromotive force in a moving wire, of the energy stream, of electromagnetic waves, and of the Hertz vibrator, do not seem to have been pointed out before, except, perhaps, in a general way, as, for example, by Boltzmann, who discusses[2] the mechanical behavior of a medium of which the equations of motion are the electromagnetic equations of Maxwell. The kinematics and dynamics of such a medium can be fairly stated only with the help of differential equations. The conceptions here developed are, of course, approximations. Some inconsistencies arise in the extension of these conceptions to three dimensions, as has been noticed by Poynting.

1. *Fundamental Conception.* — The ether is to be considered as built up of very small cells of two kinds, *positive* and *negative*, in such a way that only unlike cells are in contact. These cells are imagined to be so connected, where they are in contact, that if a cell be turned while the adjacent cells are kept stationary, then a torque, due to elastic reaction of adjoining cells, is brought to bear upon the turned cell, which tends to right it, and which is proportional to the angle turned.

2. *Conception of the Magnetic Field.* — The ether cells at a point in a magnetic field are to be thought of as rotating about axes

[1] A paper read before the Buffalo meeting of the American Association. A portion of this paper which appeared in Vol. II., Nichols and Franklin's Elements of Physics, is given here again for the sake of completeness.

[2] Wied. Ann.

which are parallel to the direction of the field at the point, the angular velocity of the cells being proportional to the intensity of the field at the point. The positive cells rotate in the direction in which a right-handed screw would be turned that it might move in the direction of the field, and the negative cells rotate in the opposite direction. This opposite rotation of positive and negative cells is mechanically possible since only unlike cells are in contact.

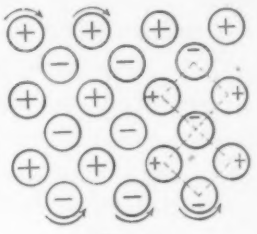


Fig. 1.

This rotatory motion of the ether cells is represented in Fig. 1. The magnetic field is perpendicular to the plane of the paper, and directed away from the reader; all the positive cells rotate clockwise, and the negative cells counter-clockwise. The kinetic energy per unit volume in such a system of rotating cells is proportional to the square of the angular velocity, which is consistent with the fact that the energy (kinetic) per unit volume in a magnetic field is proportional to the square of the intensity of the field.

3. *Conception of the Electric Field.* — The positive ether cells at a point in an electric field are to be thought of as displaced in the direction of the field, while the negative cells are displaced in the opposite direction; this displacement being proportional to the field intensity. Thus Fig. 2 represents the case in which the positive cells have been displaced towards the bottom of the page relatively to the negative cells. Figure 3 represents two meshes. The downward displacement of the positive cells has distorted these meshes which are normally square. Since this cell structure of the ether is elastic, as explained in Art. 1, its distortion, as represented in Figs. 2 and 3, represents potential energy. The amount of potential energy per unit volume is pro-

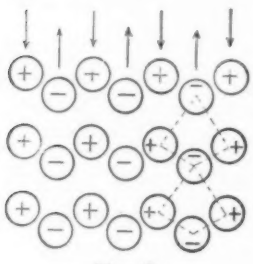


Fig. 2.

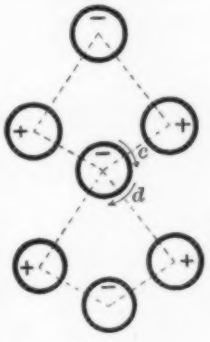


Fig. 3.

portional to the square of the displacement. This is consistent with the fact that the energy (potential) per unit volume in an electric field is proportional to the square of the field intensity.

Remark. — The two positive cells to the right of the middle cell in Fig. 3 being displaced downwards, may be conceived to exert torques upon the middle cell as shown by the arrows c and d; which torques are proportional to the intensity of the electric field, *i.e.* to the displacements of the cells. The cells to the left exert equal but opposite torques upon the middle cell.

4. *The Dependence of Electromotive Force upon Changing Magnetic Flux; Explanation of Induced Electromotive Force.* — The electromotive force around a closed curve is proportional to the rate of change of the magnetic flux through the curve.

Consider a closed boundary $ABCD$ (Fig. 4) in an electric field parallel to DA and CB. Let f be the intensity of the field along CB, and f' the intensity along DA, and let l be the length of DA and of CB. The electromotive force around the boundary $ABCD$ is $lf' - lf$. Now, the total torque acting across BC on the enclosed cells is proportional to l and to f, and the total torque acting across AD is proportional to l and to f', but opposite in direction. Therefore the total torque acting to turn the enclosed cells is proportional to $lf' - lf$. The enclosed cells gain angular velocity under the action of this torque at a rate which is directly proportional to the torque, and inversely proportional to the number of cells which participate, so that the *product* of number of cells (area of $ABCD$) into angular acceleration (rate of change of magnetic field) is proportional to the torque or to the electromotive force around $ABCD$, and this product, area of $ABCD$ into rate of change of magnetic field, is equal to the rate of change of magnetic flux through $ABCD$.

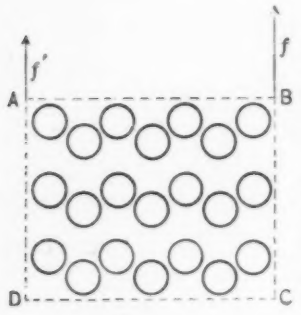


Fig. 4.

5. *The Energy Stream in the Electromagnetic Field; Preliminary Statement.* — Consider three gear wheels A, B, C (Fig. 5). Let A and C exert equal and opposite torque actions upon B.

Then, if the wheels are turning, work will be transmitted from A to C, or from C to A, according to direction of turning and to direction of torque action, and the rate of transmission of work will be proportional to the product of torque action into speed.

Fig. 5.

Imagine the cells in Fig. 2 to be rotating, positive cells in one direction, negative in the other, about axes perpendicular to the paper. This constitutes a magnetic field perpendicular to the electric field, which is towards the bottom of the page. On account of the torque actions between the cells, as explained in Art. 3, energy will be transferred to the right (or left) by each chain of geared cells at a rate which is proportional to the product of the intensity of the magnetic field into the intensity of the electric field, and the energy per second flowing across an area perpendicular to both electric field and magnetic field is proportional to the product of the respective field intensities into the area; for this area is proportional to the number of rows of cells which are acting as chains of gear wheels. The *energy stream*, that is, *energy per unit area per second*, is therefore proportional to the product of magnetic and electric field intensities, and is at right angles to both. In case the electric and magnetic fields are not orthogonal, the energy stream is proportional to the vector part of this product.

6. *The Electric Current.* — Consider a wire AB (Fig. 6) along which an electric current is flowing. The magnetic field on opposite sides of AB is in opposite directions, so that positive ether cells at p and p' are rotating in opposite directions, as shown. Since an electric current may be maintained for an indefinite time, this opposite rotation of positive ether cells on the two sides of AB cannot be due to an ever-increasing ether distortion (the cells are geared together, as it were), but there must be a *slip* between adjacent cells somewhere between p and p'. This *slip* between

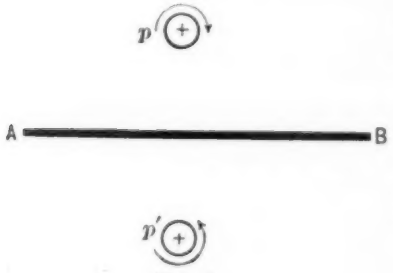


Fig. 6.

adjacent ether cells (positive and negative cells) takes place in the material of the wire, and constitutes the electric current.

7. *Established Electric Currents flow in Closed Circuits.* — Let *AB* (Fig. 7) be a wire carrying an established electric current. If this wire does not form a closed circuit, the opposite rotations of like ether cells on opposite sides of *AB* cannot continue without adjacent cells slipping on each other somewhere along any line passing around the end of *AB*.[1] That is, established *lines of slip* of the ether cells are necessarily closed lines. When a current does flow in a circuit which is not closed, an increasing ether distortion (electric field) is produced around the end portions of the circuit, which distortion produces (constitutes) electric charge there. Compare Article 12 below.

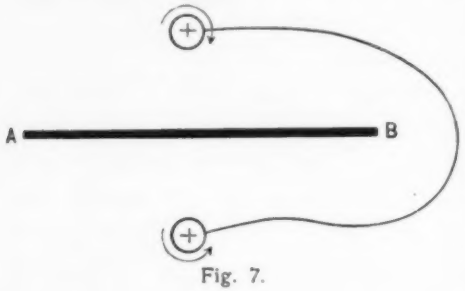


Fig. 7.

8. *Flow of Energy in the Neighborhood of an Electric Current.* — Let Fig. 8 represent the neighborhood of a long wire *AB* through which electric current is flowing. The electric field in the neighborhood is parallel to the wire, and the magnetic field circles round the wire. The product of magnetic field intensity into electric field intensity is the energy stream, and this is directed towards the wire from all sides. This energy streaming in upon the wire changes into the heat which appears in the wire. In case the wire is of high resistance, the electric field (volts per centimeter) is of great intensity, and, for the same current and same intensity of magnetic field,

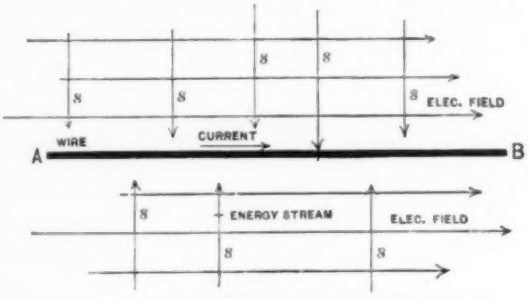


Fig. 8.

[1] The figure is thought to represent a flow of current in two dimensions.

the energy stream is correspondingly intense, making the wire very hot. At points not very near to the wire the electric field is more nearly perpendicular to the wire, especially near the battery or dynamo which is maintaining the current. The energy therefore streams out from the battery or dynamo through the whole region *surrounding* the wires, and the energy stream turns in upon the wire throughout its length.[1]

9. *The Charge on a Condenser and its Disappearance when the Condenser Plates are connected by a Wire; Preliminary Statement.* — Consider a row of gear wheels A to B (Fig. 9). If the wheel B is held stationary while A is turned in the direction of the arrow, the wheels will arrange themselves as shown in Fig. 10, alternate wheels being displaced upwards, and the intermediate wheels being displaced downwards. Conversely, a number of geared wheels, which by elastic action of any kind tend to stand in a straight row,[2] will be *relieved* from such a distortion as is represented in Fig. 10, by allowing freedom of rotation, or *permitting slip anywhere across the row.*

Fig. 9.

Fig. 10.

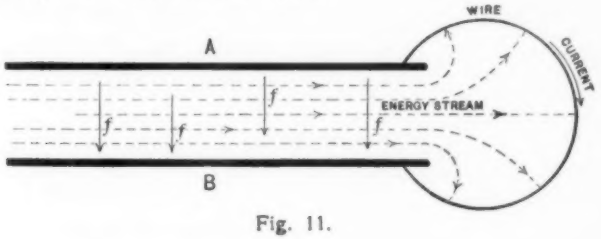


Fig. 11.

Let A and B (Fig. 11) be the plates of a charged condenser. The electric field f between these plates consists of upward dis-

[1] See J. H. Poynting, Phil. Trans.

[2] The rows of positive and negative ether cells are thought of as standing in a zigzag line, as shown by the horizontal rows in Figs. 1 and 2. In order that the diagrams may be simpler, the rows of positive and negative cells are hereafter to be thought of as straight (or uniformly curved) when free from distortion.

placement of positive ether cells and downward displacement of negative ether cells.

The horizontal rows of cells are distorted as shown in Fig. 10, and if a wire is connected across from A to B anywhere, this will constitute a line of slip permitting the adjoining ether cells to rotate, and thus entirely relieving the distortion which constitutes the field between the plates. The potential energy of the ether distortion will be transmitted along the rows of geared cells, in the direction of the dotted arrows, to the wire, where it will appear as heat.

An electric spark is a line of slip produced by the breaking down of the mechanism which sustains the electric stress, and the electric energy flows in upon a spark as it does upon a wire carrying current.

The explanation here given of the entire relief of the electric stress between two plates by the establishment of a conducting line (line of slip) between them applies to two adjacent oppositely charged bodies of any shape.

10. *The Dependence of Magnetomotive Force upon Changing Electric Flux.* — The magnetomotive force around any closed curve is proportional to the rate of change of the electric flux through the curve.

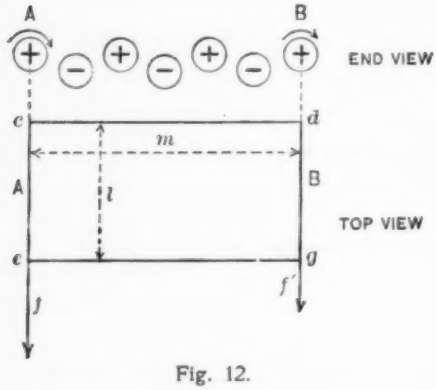


Fig. 12.

Consider a distorted chain of cells AB (Fig. 12). The rate at which this distortion is increasing is proportional to the difference in the angular velocities of the cells (intensities f and f' of the magnetic field) at A and B divided by the number of cells in the chain (distance m from A to B). Therefore, the product of the rate of increase of this distortion into the area lm of *cdeg* (rate of increase of the electric flux through *cdeg*) is proportional to $lf - lf'$, which is the magnetomotive force around *cdeg*.

11. *Production of Electric Field in the Neighborhood of a Conductor moving across a Magnetic Field; Induced Electromotive Force in a Moving Wire.* — Consider a conductor A (Fig. 13), which is moving at velocity v across a magnetic field of intensity f perpendicular to the paper and towards the reader. This body being a conductor, the ether inside of it cannot sustain electric stress, so that the energy of the magnetic field, which is seated in the region immediately in front of A, cannot be geared across A, but an electric field is called into existence about the ends of A as indicated by the lines of force, and the magnetic energy is geared around A as indicated by the lines marked *energy stream.*

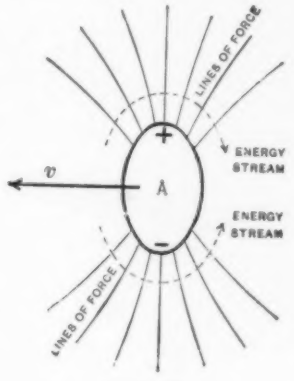


Fig. 13.

In order to explain the equation $E = lfv$, expressing the electromotive force induced in a wire of length l, moving at velocity v across a magnetic field of intensity f, we must consider the following:

Preliminary Statement. — Let the line AB (Fig. 14) be a line of force[1] of an electric field, and imagine the whole region about AB to be a uniform magnetic field perpendicular to the paper and of intensity f. The energy stream, R, at Δs, is normal to Δs and proportional to the product of the intensities of the electric and magnetic fields at Δs; that is, $R = \kappa fe$, where κ is the proportionality factor and e is the intensity of the electric field at Δs.

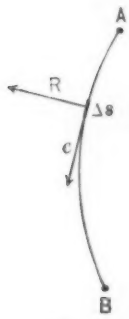


Fig. 14.

The energy per second crossing Δs (unit depth) is $R\Delta s = \kappa fe \cdot \Delta s$, and the energy per second crossing the entire line AB is $\Sigma \kappa fe \Delta s$, or $\kappa f \Sigma e \cdot \Delta s$, but $\Sigma e \cdot \Delta s$ is the electromotive force along AB. Therefore, *the energy per second crossing AB is equal to the product κfE, where E is the electromotive force along AB.*

[1] Chosen as a line of force for the sake of simplicity.

Let AB (Fig. 15) be a rod[1] of length l, moving at velocity v across a magnetic field of intensity f perpendicular to the paper.

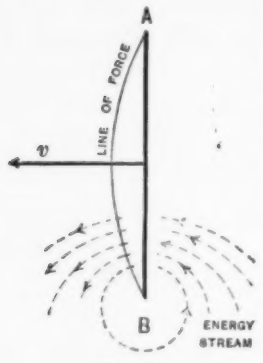


Fig. 15.

The magnetic energy $\frac{1}{8\pi} f^2 lv\Delta t$, which resides in the space, $l \cdot v\Delta t$ (unit depth), which is swept over by the rod during Δt must be equal to the energy $\kappa f E \cdot \Delta t$, which streams across the *line of force* during that interval. That is, $\frac{1}{8\pi} f^2 lv\Delta t = \kappa f E \Delta t$, or $E = \frac{1}{8\pi\kappa} lfv$. By properly choosing the unit electromotive force, we may provide that the factor $\frac{1}{8\pi\kappa}$ be equal to unity, in which case we have $E = lfv$. Further, when $\frac{1}{8\pi\kappa} = 1$, then $\kappa = \frac{1}{8\pi}$, so that the energy stream at a point in an electromagnetic field is $R = \frac{1}{8\pi} fe$, in which f and e are the intensities in "electromagnetic" units of the magnetic and electric fields at the point. R, f, and e are of course orthogonal.

When the rod AB (Fig. 15) is a portion of a closed conducting circuit, then electric stress is actually produced throughout the

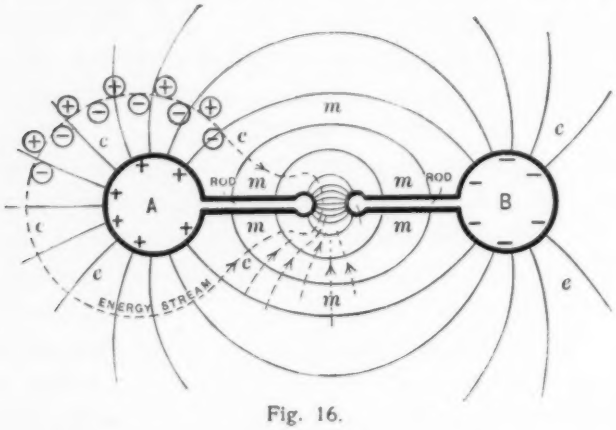


Fig. 16.

[1] The rod is assumed to be of unit width perpendicularly to plane of paper for simplicity. The formulæ given, however, are true whatever the width, for such energy as is in front of the rod does not leave the plane of the paper as it streams around the ends of AB. Such energy streams as exist in planes above and below the plane of the paper are *re-entrant* streams.

conductor as the magnetic energy streams across it, and it is the continual *slipping of gears*, or breaking down of this maintained electric stress, which constitutes the electric current in the circuit.

12. *The Action of the Hertz Vibrator* is as follows:

Let *AB* (Fig. 16) be the balls of the vibrator upon which electric charge has been collecting. Consider a chain of cells which, when undistorted, lies along the dotted line *c*, everywhere perpendicular to the lines of force. When *A* is positively charged, this chain is distorted as shown (in part), but since it is a closed chain, this distortion is fixed. When a spark occurs at the gap, a line of slip is established across this chain, and the distortion disappears as explained in Art. 9.

If the slip takes place with great friction (high electrical resistance in the gap), the cells at the spark begin turning slowly, and the entire energy of the electric field is geared into the gap and changed to heat. If the slip is almost frictionless (low electrical resistance), the electrical energy is used mostly in overcoming the inertia of the cells as they are set rotating, and after a very short interval of time a very large part of the electrical energy will have been converted into kinetic energy of the rotating cells (magnetic energy). During this conversion the energy, streaming along the dotted line, largely disappears from the regions *ee*, and is distributed mainly in the region *mm*. When the chain of cells has been freed from distortion, the rotatory motion of the cells between *A* and *B* will have reached a maximum, and on account of their momenta the cells will continue turning, and produce a distortion of the chain in a reversed sense. At the same time the energy will, to a large extent, stream back from the region *mm* to the regions *ee*, the ball *A* will be negatively charged, and the ball *B* will be positively charged. This reversed distortion of the chain of cells is then relieved by a reversed slip (a reversed current in the rods and gap), and so on.

These oscillatory changes take place so rapidly that the portions of the distorted ether which are remote from *AB* do not follow the changes promptly. This gives rise to electrical waves, the nature of which at a distance from the vibrator is explained in the following article.

13. *Electromagnetic Waves.* — Imagine the ether cells between the dotted lines A and B (Fig. 17) to be rotating (magnetic field perpendicular to paper). There will soon be an elastic distortion [electric field towards top (or bottom) of page] between these rotating cells and the stationary cells to right (or left). This elastic reaction will soon stop the motion of the cells between A and B and set the adjoining cells in motion. These will in turn act upon the cells next beyond, and so forth. Thus the original layer AB of the magnetic field will give rise to two waves, to right and left, and these waves will consist of mutually perpendicular electric and magnetic fields. If the cells between A and B (Fig. 17) are, to begin with, displaced upwards (figure represents positive cells only), their reaction upon adjoining cells will start rotatory motion, and the result will be two waves, to right and left, as before.

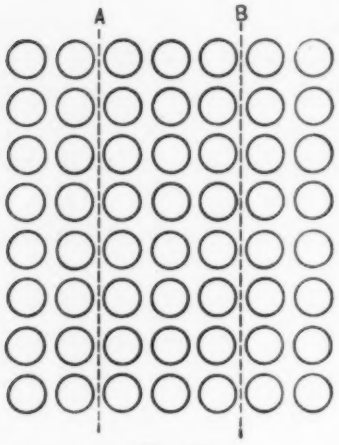


Fig. 17.

14. *Stationary Electromagnetic Wave Trains; Reflection with and without Change of Phase.* — Consider the row of cells AB (Fig. 18) along which a train of electromagnetic waves is approaching a conducting wall W. Imagine the wall to be a perfect conductor, for simplicity; then the wave train is totally reflected, and the reflected train superposed upon the incident train gives a stationary train. The disturbance at any point is the superposition of the disturbances at that point due to the incident and reflected trains respectively. Now at the face of the wall the electric field must be zero (the ether may be imagined to end here, and the freedom of motion of the end cell prevents elastic distortion between that cell and the next), so that the electric intensities of

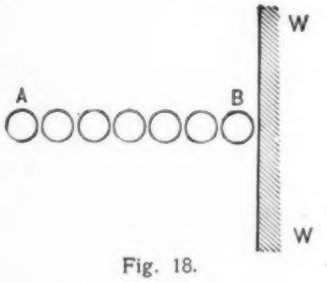


Fig. 18.

the incident and reflected trains must be continually equal and opposite at the wall. This is reflection with change of phase of electric intensity (without change of phase of magnetic intensity).

If the wall W be imagined to have infinite magnetic permeability (indefinitely great moments of inertia of ether cells), then the magnetic intensity at the wall would necessarily be zero, so that the magnetic intensities of the incident and reflected trains must be continually equal and opposite at the wall. This is reflection with change of phase of magnetic intensity (without change of phase of electric intensity).

In a stationary electromagnetic wave train the energy streams from regions near the nodes (electric) to regions in the vibrating segments (electric) and back again, in a manner very similar to the back-and-forth flow of the energy in the region surrounding a Hertz oscillator.

MINOR CONTRIBUTIONS.

ON A POSSIBLE DEVELOPMENT OF THE IDIOSTATIC ELECTROMETER.

BY C. BARUS.

1. The idiostatic electrometer has not hitherto been developed to an extent comparable with the quadrant electrometer, at least in so far as measurement of small potential differences is concerned. Yet the idiostatic method is apparently the more reasonable, seeing that it introduces no foreign potentials to coöperate with those under investigation. It might seem possible, moreover, to secure an advantage by dispensing with the viscous liquid in which the swinging needle of the quadrant electrometer is damped. I purpose, therefore, in the following paper to describe certain results obtained in an endeavor to perfect the idiostatic electrometer. The experiments proved to be delicate; indeed, one needs to cope with the trying difficulties which I have met with in these attempts to really appreciate the admirable efficiency of the quadrant electrometer.

2. The two forms of apparatus available are the gold-leaf type and the condenser type, but only the latter gave me results which seem worth describing. The pattern adopted is identical with Lord Kelvin's[1] absolute

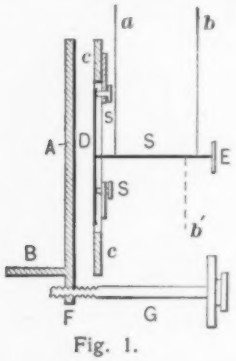


Fig. 1.

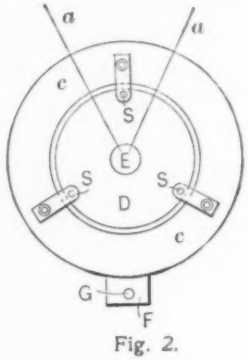


Fig. 2.

electrometer, and the conditions of sensitiveness are therefore given. To fix the ideas, let the considerations refer to the apparatus annexed, as this is a diagrammatic view of the one used below. Here *A* is the fixed, and *D* the movable, plate of the condenser, *c* the guard ring, Fig. 2 being a

[1] See papers on Electrostatics and Magnetism, p. 281 *et seq.*

front view, Fig. 1 a section at right angles to it. If the disk is to be movable parallel[1] to itself, it may be suspended from parallel silk threads a and b, each of which is broadly bifilar above (see Fig. 1) to prevent lateral oscillation. The disk D is made very light, and the guard ring closed behind in form of a shallow box, with a small central hole at d to admit of free motion of the stem S. Three small screws s serve as stops for the disk D. Finally, the plate A is free to slide on B parallel to itself, and this motion is controlled by the micrometer screw G working in the lug F. E is a small, thin-plate mirror rigidly connected with the movable disk D by the stem S. If the system DSE, with the broad disk D, be made very light, the air damping is excellent.

Suppose, now, that the difference of potential between A and D in this apparatus is 1 volt, and their distance apart d. Then, if A be the area of D in square centimeters, and F the total force on the disk in dynes,

$$F = A/(8\pi \cdot 10^4 \cdot 9\,d^2).$$

If $A = 8\pi$, or if the area of A be made 25.133 cm^2. (diameter 5.65 cm., a convenient size in practice), we have, simply, $F = (300\,d)^{-2}$; or

$d = 1.00$ cm.	$F = 0.000011$ dynes
$= 0.10$	$= 0.001100$
$= 0.01$	$= 0.110000$

Thus the forces to be measured are excessively small even for two disks as near together as $\frac{1}{10}$ mm.

In the above apparatus, forces to equilibrate these values are, for small deflections, $mg \cdot \phi$, where m is the mass of the system DSE, and ϕ the deflection of the pendulum. If l be the length of the pendulum, and x its displacement, $\phi = x/l$; and therefore $x = Fl/mg$.

The value for l being 22.5 cm., the mass of the disk 0.5 g., and its appurtenances 0.71 g., the excursions x for different distances apart d of the condenser plates A and D are:

$d = 1.00$ cm.	$x = 0.0000003$ cm.
$= 0.10$	$= 0.0000352$
$= 0.01$	$= 0.0035200$

The excursions x to be measured are thus correspondingly small, and, for $d > 0.1$ cm., small even as compared with the wave length of light.

[1] If rotation of the disk around the horizontal is to accompany the motion, threads a and b' may replace a and b. The mirror E may then be observed with scale and telescope.

3. Having recently had occasion to work with Michelson's refractometer,[1] it seemed worth investigating whether the method could be used for a movable mirror E under the above conditions. The excellent damping secured for the disk D had made this probable. The adjustments are given in the diagram (Fig. 3), where E' is the stationary plane mirror corresponding to the mirror E of the system DSE, P the thick refractometer plate, and T the telescope. The sodium flame and appurtenances are placed beyond L. The optical conditions are very fully discussed in Michelson's great memoir (*l.c.*).

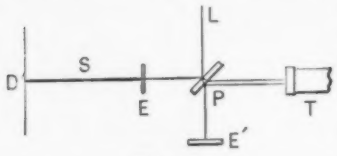


Fig. 3.

I found it convenient to use the large, flat plate of an old Kirchhoff spectrometer, the above electrometer ADE replacing one of the telescopes, and the mirror E' the other. The glass plate P on leveling screws was moved on the plate. E' being revoluble on the axis of the spectrometer, it required fully four complete rotations of the micrometer tangent screw (about 0.4°) to pass the fringes from the initial to the final vanishing point through maximum visibility.

Thus far I have not obtained results with a *free* disk D. The fringes so observed were fleeting; they shifted, rotated, and changed form continually when visible, and reappeared too rarely. But I have not given the matter up, as I think that, with better provision against air currents and earth tremors, the attempt will be feasible. For the present, I added stop screws as shown in the figure at s. These kept the fringes in the field, but did not quite wipe out their slow and regular motion. With a rather heavy disk (3.5 g.) of tin plate, and a distance apart d of about 0.25 cm., I obtained a deflection of one fringe per 20 volts of potential difference between A and D. For a free disk, the excursion x should have corresponded to about fifteen fringes for 20 volts; or we would have obtained fifteen times as much displacement had it not been necessary to lift the disk off the stops.

4. The plan for a feasible apparatus has thus been suggested; viz. to allow the disk to recline very lightly on stops (s, s, s, Fig. 1), and to move the plate A forward by the micrometer screw until the disk is just lifted off by a fraction of a wave length. Then the difference of potential is simply proportional to the distance d between the plates.

Before deciding to make a definite piece of apparatus with a micrometer slide, I wished to test the present arrangement further with the object of ascertaining how far the sensitiveness could be pushed. Accordingly in

[1] Michelson, "Valeur du Mètre en Longueurs d'Ondes Lumineuse," Tome xi., Trav. et Mém. du Bureau internat. des Poids et Mesures, 1894.

the following experiments the face F is still reckoned in terms of the number of wave lengths which pass the cross-hair in the telescope when the condenser is charged and discharged alternately.

To obtain a sensitive system, the disk D was made of thin plate mica, silvered on both sides. The mass of the movable system DSE was thus reduced to 0.71 g. It was charged through the stop screws s. In operating the apparatus I encounted a formidable difficulty, inasmuch as with a very light disk all but free, there was *no contact* between the disk and the stops, and it was now, as a rule, found impossible to charge the disk with small potentials. Liquid contacts, aside from other reasons, are evidently out of the question because of the relatively immense capillary forces involved. After many trials I finally found that a sure contact could be maintained between the guard ring and the disk by connecting the two permanently with a long narrow pendent strip of gold leaf (not shown in Figs. 1 and 2). The ends of this were attached with mucilage to the guard ring C, and the disk D, in such a way as not to hamper the motion. Again, because of the difficulty of obtaining contacts[1] with small voltages it was now possible to move the disk D almost indefinitely near A, and thus obtain relatively great sensitiveness, without the risk of discharging the condenser. Finally, in view of the close proximity in question, the motion of the disk was so slow, that there was no difficulty in counting the fringes on charging or discharging.

A trial of the apparatus in the new form gave me much better results, the fringes passing for a single volt being about 10. Since in an undisturbed apparatus less than $\frac{1}{10}$ fringe may be estimated, the sensitiveness has thus become about $\frac{1}{100}$ volt. Had the disk been about 0.05 cm. from the opposed condenser plate, about 5 fringes would have passed. Hence the disk must have been very near the plate, and the stop screws in good adjustment. Many experiments of this kind were made, but they yielded no additional results. Most of the work had to be done at night.

5. In conclusion it is interesting to inquire into the construction and sensitiveness of divers apparatus of the present kind under favorable conditions. It would be best to make the plate A and guard ring C of plate glass with rounded outer edges and silvered on both sides. The disk D, if made of microscope cover glass, silvered, would not weigh above 0.035 g. per square centimeter, or about 0.88 g. for the area 8π cm^2., which is within one gram for the movable system of disk and mirror. In such a case, parallelism between the silvered plates could be tested in the usual optic way by reflection, and the adjustment of the set screws controlled by

[1] The usual absence of contact here recalls the case of two soap bubbles which fail to adhere unless pressed together for some time, or otherwise brought in contact. Cf. E. Kaiser, Wied. Ann., LIII., p. 667, 1894.

their own reflected images. If with better protection against air currents, etc., it is possible to work with a free disc D, the electrometer would remain absolute in its registry. The sensitiveness of such an apparatus would probably be of the order of 0.001 volt for a minimum of dielectric thickness.

If the absolute character of the instrument be sacrificed in favor of greater sensitiveness, one may profit by the above experience, and replace the movable system DSE by a sheet of gold leaf made to cover the whole space on the inner face of the guard ring smoothly. The mirror E could then be reduced in size so as to weigh not much above a milligram. In such a system the counter force would be in part flexure and in part gravity. The former may be evaluated by the aid of the equation

$$5\,x = (r^4/s^3)\,(p/E),$$

where x is the central displacement due to the pressure of p dynes per square centimeter, r the radius of the disk (2.8 cm.), and s its thickness (say 0.0001 cm.). The pendulum forces are found as above. The results show that in this case $x = F/4$ nearly, where F is the total force on the disk, as given in § 2. Hence such an instrument should in the extreme case register as small a potential as 100 microvolts, supposing $d = 0.01$ cm., and the motion of one-tenth of an interference fringe discernible.

Finally, the best conditions are secured by measuring the counterforces not by gravity, but in the usual way, by torsion. I have made an estimate based on the above experiments, supposing that two identical disks like D, in Fig. 1, are symmetrically joined by a light rigid rod, 20 cm. long, and suspended from a fine wire, 0.01 cm. in diameter, attached to the middle of the rod, with the ends fixed 10 cm. above and below it. In such a case, if $d = 0.01$ cm., the motion of one-tenth interference fringe would correspond to less than 20 microvolts. Whether this sensitiveness can be attained or not will depend very largely on quiet surroundings. Beyond this there does not seem to be more serious difficulty than was encountered in the above experiments carried on with very ordinary facilities.

BROWN UNIVERSITY, PROVIDENCE, R.I.

EMPIRICAL FORMULÆ FOR VISCOSITY AS A FUNCTION OF TEMPERATURE.

BY A. WILMER DUFF.

SEVERAL formulæ for the viscosity of liquids at different temperatures have been proposed. The writer has recently required the use of such a formula for a wide range of variation of viscosity. The following

classification and discussion of the different formulæ tried, together with the suggestion of a generalized formula, may be of interest. Besides its general importance as a means of interpolation, extrapolation, and tabulation, the viscosity formula at present derives some special interest from the attempt which Stoel, Heydweiller, Thorpe and Rodger, and others have made to use it for studying the relation of chemical constitution to physical properties.

A	$\eta = \frac{C}{1 + at + bt^2}$	(Poiseuille,[1] Meyer).
B	$\eta = a + bt + ct^2 + dt^3$	(Koch,[2] Wagner[3]).
C	$\eta = \frac{C}{1 + at}$	(Meyer, Rosencranz[4]).
D	$\eta = C\frac{a - t}{t - \beta}$	(Grätz[5]).
E	$\eta = \frac{C}{a + t} - b$	(Slotte[6]).
F	$\eta = \frac{C}{(a + t)^n}$	(Slotte, Thorpe and Rodger[7]).
G	$\eta = Ce^{-at}$	(Stoel, Heydweiller,[8] Reynolds[9]).

Formula *A* has been chiefly used for water, *B* for mercury; both of them are unsuitable for a wide range of variation of viscosity. *C* is a particular case of *E*; *D* was deduced theoretically by Grätz, and applied to a large number of substances, but it will not represent alcohols, ethyl ether, water, etc. (see also Thorpe and Rodger). The remaining formulæ, *E*, *F*, *G*, are independent of one another, and for each a considerable degree of generality may be claimed; yet the curves they represent diverge considerably from the viscosity curve. Thus *E* and *G* give values that are too small at both high temperatures and low temperatures; while *F* gives values that are too great at high temperatures and too small at low temperatures. Thorpe and Rodger in their very extensive and careful work on about seventy different substances have, after trying other formulæ, chosen Slotte's *F*; but its insufficiency for substances of wide variation of viscosity, such as isobutyl alcohol, amyl alcohol, dimethyl ethyl carbinol, and also for water, is so great that in each case they divide the whole viscosity curve into three parts (for water two) and apply a separate formula of

[1] Pogg. Ann., LVIII. (1843).
[2] Wied. Ann., XIV. (1881).
[3] Wied. Ann., XVIII. (1883).
[4] Wied. Ann., II. (1877).
[5] Wied. Ann., XXXIV. (1888).
[6] Wied. Ann., XIV. (1881).
[7] Phil. Trans., Vol. 185 (1894).
[8] Wied. Ann., LV. (1895).
[9] Phil. Trans., Vol. 177 (1886).

Slotte's form F to each part. Thus in the case of amyl alcohol the three equations call for the calculation of nine constants from the twelve determinations made, leaving only three values to be represented non-identically. In such cases the value of the formulæ for the purposes above enumerated seems small.

TABLE I.

VISCOSITY OF ISOBUTYL ALCOHOL.

Temperature.	Observed.	Calculated from F (thrice applied).	Per cent error of F.	Calculated from H_2.	Per cent error of H_2.
0.45	0.079111	0.079111	0.0	0.079313	0.2
9.90	0.055735	0.055250	0.9	0.055609	0.2
19.01	0.039779	0.040285	1.2	0.040705	2.3
27.77	0.030658	0.030439	0.7	0.030658	0.0
38.16	0.022398	0.022392	0.0	0.022367	0.1
47.44	0.017217	0.017212	0.0	0.017233	0.3
56.48	0.013571	0.013549	0.1	0.013526	0.3
56.59	0.013502	0.013511	0.1	0.013488	0.1
65.95	0.010697	0.010711	0.1	0.010694	0.0
74.61	0.008748	0.008748	0.0	0.008748	0.0
83.95	0.007173	0.007160	0.2	0.007143	0.4
93.85	0.005864	0.005868	0.1	0.005847	0.3
105.07	0.004753	0.004753	0.0	0.004740	0.3

$$\ln H_2 \begin{cases} a = 385.88 \\ \alpha = 0.0074052 \\ \beta = 57.92 \\ C = 0.88930 \end{cases}$$

To compare E, F, and G, let us differentiate the logarithms of each side and invert.

$$\text{From } E \text{ (or } D) \quad -\eta\frac{dt}{d\eta} = C_1(a-t)(t-\beta). \qquad E'$$

$$F \qquad -\eta\frac{dt}{d\eta} = C_2(a+t). \qquad F'$$

$$G \qquad -\eta\frac{dt}{d\eta} = C_3. \qquad G'$$

Now, since $-\eta\frac{dt}{d\eta}$ is the subtangent of the viscosity curve, let us call the curve got by plotting subtangents against temperatures the "subsidiary curve of subtangents" (or briefly, the "subsidiary curve"). Then we see

from E', F', G' that we may interpret E, F, G as meaning, respectively, that the subsidiary curve is a parabola cutting the line of temperatures in real points, a straight line inclined to the line of temperatures, a straight line parallel to the line of temperatures. To test this point I have, for several substances, obtained the subtangents of the viscosity curve by the direct application of a straight-edge, and so plotted the subsidiary curve. This I find to be a parabola, convex to the axis of temperatures, and cutting that axis either in real points (as in the cases of water and most

TABLE II.

VISCOSITY OF DIMETHYL ETHYL CARBINOL.

Temperature.	Observed.	Calculated from F (thrice applied).	Per cent error of F.	Calculated from H_2.	Per cent error of H_2.
0.49	0.137969	0.137969	0.0	0.137820	0.1
9.31	0.082034	0.080650	1.7	0.080772	1.5
18.48	0.049978	0.050067	0.2	0.050090	0.2
27.24	0.033643	0.033643	0.0	0.033643	0.0
36.42	0.023322	0.023278	0.2	0.023276	0.2
45.05	0.017135	0.017133	0.0	0.017120	0.1
53.18	0.013199	0.013204	0.0	0.013203	0.0
62.95	0.009943	0.009943	0.0	0.009976	0.3
71.91	0.007931	0.007935	0.0	0.007921	0.1
81.06	0.006400	0.006418	0.3	0.006400	0.0
89.94	0.005301	0.005303	0.0	0.005302	0.0
95.70	0.004718	0.004718	0.0	0.004734	0.3
96.70	0.004643	0.004625	0.4	0.004660	0.3

$$\text{In } H_2 \begin{cases} a = & 3.0860 \times 10^8 \\ \alpha = & 0.025365 \\ \beta = & 101.10 \\ C = & 2.14746 \times 10^9 \end{cases}$$

substances of slow variation of viscosity), or in imaginary points (as in the cases of glycerin, amyl and isobutyl alcohols, dimethyl ethyl carbinol, mercury, and most substances of rapid variation of viscosity). The general equation of the subsidiary curve is therefore

$$y = \left[-\eta \frac{dt}{d\eta} \right] = a + bt + ct^2. \qquad H'$$

For particular values of a, b, c this will, of course, include E', F', G'. Hence, if we integrate H', we must obtain a formula of a sufficiently gen-

eral type to represent all liquids representable by E, F, or G, and also liquids which give, as a subsidiary curve, a parabola not cutting the temperature axis in real points. This latter class seems to include those

TABLE III.

VISCOSITY OF WATER.

Temperature.	Observed.	Calculated from F (twice applied).	Per cent error of F.	Calculated from H_1.	Per cent error of H_1.
1.13	0.01709	0.01712	0.2	0.01714	0.3
3.09	0.01607	0.01609	0.1	0.01605	0.2
4.47	0.01535	0.01535	0.0	0.01535	0.0
5.45	0.01494	0.01494	0.0	0.01488	0.4
13.53	0.01181	0.01179	0.2	0.01177	0.4
22.02	0.00955	0.00951	0.4	0.00951	0.4
30.72	0.00786	0.00784	0.2	0.00785	0.1
39.32	0.00662	0.00662	0.0	0.00662	0.0
47.03	0.00576	0.00577	0.2	0.00577	0.2
55.53	0.00501	0.00502	0.2	0.00502	0.2
64.01	0.004415	0.00442	0.1	0.00442	0.1
72.54	0.003915	0.00393	0.4	0.00393	0.4
80.75	0.003525	0.003535	0.3	0.00353	0.1
89.90	0.003165	0.003165	0.0	0.003165	0.0
98.09	0.00289	0.00289	0.0	0.00289	0.0
100.00	0.00283	0.00283	0.0	0.00283	0.0

$$\text{In } H_1 \begin{cases} \alpha = 6005.52 \\ \beta = 45.28 \\ n = 1.6012 \\ C = 7.1179 \times 10^{-6} \end{cases}$$

substances of rapid variation of viscosity which have hitherto not been reducible to any empirical formula.

To avoid imaginaries, the integrated formula is expressed in two ways. (1) When the subsidiary curve cuts the temperature axis in real points, and when, therefore, in H', $b^2 > 4ac$,

$$\eta = C\left(\frac{t+\alpha}{t+\beta}\right)^n. \qquad H_1$$

(2) When the subsidiary curve does not cut the temperature axis in real points and when, therefore, in H', $b^2 < 4ac$,

$$\eta = Ca^{-\tan^{-1}\alpha(t+\beta)}. \qquad H_2$$

H_1 is of course a generalization of E and F. For let $n = 1$ and we get E. Again write H_1 thus,

$$\eta = \frac{C\alpha^n}{(t+\beta)^n}\left(1+\frac{t}{\alpha}\right)^n,$$

and let α be supposed infinitely large while n and $C\alpha^n$ remain finite, and we get F. Hence all the substances represented by Rosencranz, Grätz, Slotte, Thorpe and Rodger, and others by means of C, D, E, or F are representable still more perfectly by H_1. Again H_2 includes G as a particular case; namely, that in which β is very small. Hence H_2 will cover the results of Stoel, Heydweiller, Reynolds, and others.

TABLE IV.

VISCOSITY OF MERCURY.

Temperature.	Observed.	Calculated from B.	Per cent error of B.	Calculated from H_2.	Per cent error of H_2.
21.4	0.01868	0.01847	1.13	0.01865	0.1
0.0	0.01698	0.01697	0.00	0.01698	0.0
9.2	0.01631	0.01640	0.60	0.01634	0.2
10.1	0.01620	0.01631	0.70	0.01628	0.5
12.5	0.01599	0.01618	1.18	0.01612	0.7
18.3	0.01561	0.01582	1.20	0.01568	0.5
98.8	0.01228	0.01224	0.30	0.01218	0.8
99.1	0.01225	0.01222	0.20	0.01217	0.6
154.0	0.01092	0.01090	0.20	0.01089	0.3
176.2	0.01047	0.01045	0.10	0.01052	0.5
272.0	0.009492	0.009477	0.20	0.009477	0.2
314.7	0.009183	0.009172	0.20	0.009185	0.0

$$\text{In } H_2 \begin{cases} a = 1.9165 \\ \alpha = 0.006917 \\ \beta = 34.3 \\ C = 0.01976 \end{cases}$$

Thus to completely establish the generality of H_1 and H_2 it seems only necessary to show that they apply to substances of rapid variation of viscosity, for which F is unsuitable, and also to water and mercury, which A and B, respectively, have been regarded as expressing most satisfactorily. The two substances for which Thorpe and Rodger found F to fail most signally were isobutyl alcohol and dimethyl ethyl carbinol. They will be found represented by H_2 in Tables I. and II., the approximations given by

H_2, once applied, being about as good as those given by F (Thorpe and Rodger's calculations) thrice applied. Tables III. and IV. show H_1 applied to water (Thorpe and Rodger's figures being used) and H_2 applied to mercury (Koch's figures being used).

The only point calling for further mention is the calculation of the constants in H_1 and H_2. Here we have a choice of three methods.

(1) From the viscosity curve subtangents are found directly and employed to obtain the constants of the subsidiary curve; after integration, the remaining constants are found without any difficulty.

(2) The subtangents at three or more points on the viscosity curve are found by applying at such points an approximate formula, such as E or G, and then deducing the values of $\eta \frac{dt}{d\eta}$; the remaining procedure is as before.

(3) Formulæ for the calculation of the constants in H_1 and H_2 are developed. From the viscosity curve, obtain four values of η in geometrical progression and the corresponding values of t. Then in

$$\eta_2^2 = \eta_1\eta_3,$$

$$\eta_3^2 = \eta_2\eta_4,$$

substitute the values of $\eta_1, \eta_2, \eta_3, \eta_4$, in terms of t_1, t_2, t_3, t_4, given by the formula H_1 or H_2; equations, containing only $\alpha, \beta, t_1, t_2, t_3, t_4$, will be found from which α and β may be readily deduced. I have used (1) and (3) for mercury, the better formula (used in calculating Table IV.) being found by (3). For water, isobutyl alcohol and dimethyl ethyl carbinol method (2) was employed.[1] Ways of "improving" the constants and applying the method of least squares will readily suggest themselves. I have not employed such methods; the constants were but once calculated (except in the case of mercury) and the agreement between observed values and calculated values is therefore the more striking.

The above method of constructing empirical formulæ by deriving some recognizable subsidiary curve may be novel; it seems to greatly enlarge the possibilities of finding a suitable formula, and though it may lead to complex expressions it provides its own method of determining their constants.

PURDUE UNIVERSITY, LAFAYETTE, IND.

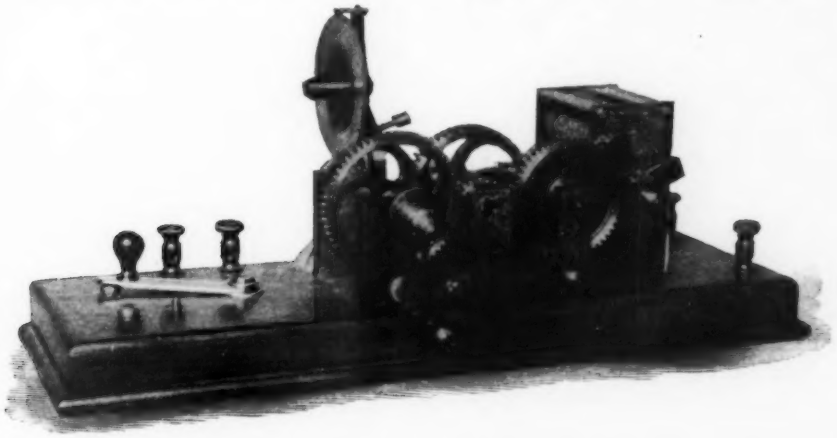
[1] In Table I. the observed value of η at 19°.01 probably contains a large error; and the attempt to include this value in formula F no doubt accounts for the large deviations of the formula at 9°.90 and 27°.77. In Table III. there is probably a large error at 9°.31.

A Synchronous Motor for Determining the Frequency of an Alternating Current.

By George S. Moler.

WHEN an alternating-current dynamo is in operation, there are as many different currents during one revolution as there are poles. One half of these will be in one direction and the remainder in the other, the complete periods during one revolution being equal to the number of pairs of currents, or in other words equal to half the number of poles; therefore the frequency which is the number of pairs per second will be the number of revolutions per minute times one half the number of poles, divided by sixty. If one wishes to determine the frequency of an alternating current when the dynamo is too distant or is inaccessible, some other method than the taking of its speed will have to be employed.

There are several ways of experimentally determining the frequency, but perhaps one of the best of these is by making use of a synchronous motor and a speed counter. A synchronous motor, as its name implies, is



one the speed of which depends upon the frequency of the alternating current which drives it. The speed of such a motor is controlled as positively as if the motor were connected by a shaft to the generator itself, the number of poles passed in a second in the motor and in the generator being the same.

The piece of apparatus described in this article consists of a synchronous motor and an electrically operated speed counter built in one piece. The current will only drive it after it has been put up to the proper speed, so a crank handle with gearing is provided for that purpose.

The motor is made by placing two magnet spools on the opposite sides of a small armature shaft and with their axes parallel to it. The spools have cores made of bundles of soft iron wires. Upon the revolving axis are placed two soft iron armatures, each having four arms. They are placed at opposite ends of the spools, and the arms of one are placed parallel to the corresponding arms of the other so that they pass both ends of both spools at the same time. On account of this relation of arms to spools occurring four times in each revolution, it requires four currents, that is, two each way, to cause one revolution of the motor. On account of the above fact, if the number of revolutions made in a minute by this shaft were taken, the frequency would be found by dividing by thirty.

In the apparatus of which a view is given, the speed counter is arranged to give the speed of the crank shaft, which is geared to run only one thirtieth as fast as the armature shaft, so the number of revolutions as read for one minute is the frequency direct without any further calculation. The worm driving the counter is carried by friction and has a ratchet wheel connected with it. There are two electro-magnets operated by a clock closing the circuit once a minute; one of these is for releasing the worm-gear at the beginning of the minute and the other to throw a catch into the ratchet wheel to stop it. Another catch will only operate to stop the worm when the wheel comes around so it reads zero. There is also a graduated circle attached to the worm and this is also stopped at zero at the same time. At the end of a minute's run the electro-magnet throws the catch into some one of the twenty teeth, thus giving the reading to five one-hundredths of an alteration.

The apparatus is small and portable, the base being 14 inches long and the weight being 8.5 pounds. The minimum current which will operate it is 1.36 amperes at about 10 volts, but 2 amperes is a better amount, as it is easier to start the motor. One 32-candle power 50-volt incandescent lamp has a suitable resistance to put in series with the motor when it is to be connected into a 50-volt lighting circuit and the current can be obtained at any 16-candle-power lamp socket usually without changing the fuses. In case a clock and battery are not convenient to attach, the armatures of the electro-magnets may be operated by hand.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
November, 1896.

Lecture-room Demonstration of Orbits of Bodies under the Action of a Central Attraction.

By R. W. Wood.

NOT remembering to have seen any attempt to show experimentally in the lecture room the motion of bodies acted on by a central attractive force varying inversely as the square of the distance in elliptic, parabolic, and hyperbolic orbits, I have made a few experiments with a view of determining how well these curves could be imitated by the motion of a small steel ball around a magnetic pole. The results were so good that I feel warranted in making them known, and believe that the experiment may be found useful in making more cheerful that portion of the course usually rather destitute of pyrotechnics.

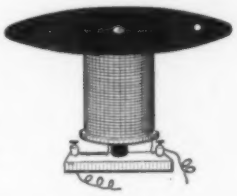


Fig. 1.

The apparatus used was very simple, consisting of a circular glass plate about 40 cm. in diameter, with a small hole in the center through which projected the somewhat conical pole piece of a large electro-magnet (Fig. 1). The surface of the plate was smoked, and it was made level as nearly as possible, the axis of the magnet being of course vertical.

A small, highly polished ball of steel about 5 mm. in diameter (from a bicycle bearing), when projected across the plate, traced its path in the soot and left a permanent record of its motion.

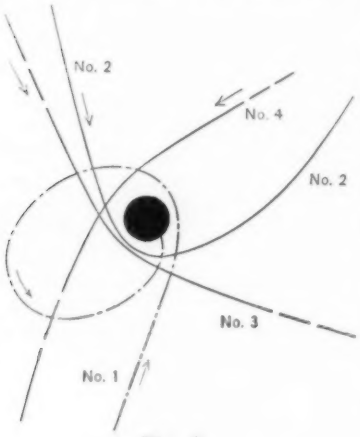


Fig. 2.

Under these conditions gravity exerts no direct influence on the motion, and we have only the initial velocity and the central attractive force to deal with, together with the loss of velocity due to friction. There are several other circumstances which make the conditions unlike those existing in the case of two gravitating bodies in space, and taking everything into consideration, it is quite surprising what good results were obtained.

The ball was blown out of a short piece of glass tubing held in the plane of the plate with varying initial velocities, and curved orbits obtained which were at least good imitations of the ellipse parabola and hyperbola.

Figure 2 is a photograph[1] of a plate showing all three forms, the black spot in the center being the hole occupied by the magnet pole; the arrows indicate the direction of the motion.

Number 1 was produced with low initial velocity, and is a very fair representation of an ellipse, with the attractive force in one focus. The loss of velocity due to friction caused the ball to "fall into the sun" after completing one revolution, a one year's existence of the system.

On another trial an ellipse (*spiral*, strictly speaking) was obtained that was almost reëntering, the miss being not more than a couple of millimeters, while in the one figured it was nearly a centimeter.

The right-hand branch of No. 2 resembles a parabola, and was produced by a somewhat higher initial velocity. It will be noticed that the ball moved to its perihelion position in a path rather like a hyperbola, and on rounding the pole, its velocity having been diminished somewhat, moved off in a parabola. It would be more exact probably if we called this curve an ellipse of great eccentricity, since the conditions governing the formation of a parabolic orbit would be difficult even to approximate.

Numbers 3 and 4 are hyperbolæ, produced by still higher initial velocities.

None of the orbits shown in the figure are as perfect as some that have been obtained by accident on other plates. It is quite difficult to make a plate showing all three forms with only four or five trials, as the velocity has to be nicely adjusted; consequently the curves shown in the figure must not be taken as samples of the best that can be produced by a large number of trials.

The hyperbola is of course the easiest to produce, and the parabola the most difficult. Some device for regulating the initial velocity and aim would be conducive to more uniform results.

Polarization of the steel ball is apt to give trouble, and I have obtained some repulsion orbits where the ball turns back before reaching the center, which are very pretty, but not desirable when one is trying to illustrate central attraction. Soft iron balls would be preferable to steel on this account, but they are not on the market so far as I know, and the others answer the purpose well enough.

[1] Figure 2 has been reproduced by the engraver from an untouched photograph. — Ed. PHYSICAL REVIEW.

The Refractive Index of Water and Alcohol for Electrical Waves.

By A. D. Cole.

In the September–October number of the Physical Review, J. F. Mohler criticises my paper on "The Refractive Index, etc., of Water and Alcohol for Electrical Waves," which was given in abstract in the preceding number. I regret that the limits of an abstract did not enable me to be perfectly clear on the points referred to. Reference to the full paper in *Wiedemann's Annalen*, Vol. 57, p. 291, 292, will show that I have *not* ignored the absorption of the medium. Reasons are there given for the belief that the absorption is very small for long waves, although very considerable for short waves. Results published by continental investigators since my work was finished strengthen this conclusion. See Cohn and Zeeman in *Wied. Ann.*, Vol. 57, p. 22, and P. Drude in the same, Vol. 54, p. 352, and Vol. 55, p. 633. Also, Ellinger was easily able to measure the deviation produced on long waves by a thick prism, while my experiments with short waves showed almost complete absorption by a much thinner prism. It was on account of this difference that I used the reflection method in my study of the short waves.

As to my finding but four bridge positions in the trough of water and two in the alcohol, these limits were set by the shortness of the trough, which was but 68 cm. long. There was no room for another bridge in either case.

The bearing of absorption on the reflection method used for the short waves is considered in a footnote on page 300, *Wied. Ann.*, Vol. 57.

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NEW BOOKS.

Higher Mathematics: A Text-book for Classical and Engineering Colleges. Edited by MANSFIELD MERRIMAN and ROBERT S. WOODWARD. 8vo, pp. xi+576. New York, Wiley & Sons, 1896.

The advent of this book marks a new step in the development of teaching facilities in higher mathematics. It is intended to do what no other single mathematical text-book does, viz. gather together in one volume brief presentations of the different extensions of higher mathematics, in order that in each branch the student may become acquainted at least with the nature of the field concerned, with the underlying considerations, with the elementary development, and with its simpler applications.

As the editors well remark in the preface, the master minds of the early part of the century could hope by industry to obtain a comprehensive if not minute knowledge of the various branches of mathematical science; but since that date so great has been the tendency toward specialization, and so extensive and detailed have been the developments in the various branches, that now the mere quantity of information available presents a formidable if not insurmountable obstacle to the attainment of the breadth of knowledge which characterized the mathematician of this earlier period. It is further pointed out, however, that these extensions have been concerned with details rather than with the development of essentially new principles, and hence that the modern student, in entering the higher ranges of mathematics, needs most of all a general guide which shall fix his attention on essentials, and prevent him from wasting time and energy in the pursuit of non-essentials. The editors also point to an existing demand for a text-book in the best classical and engineering colleges for the mathematical work following the usual course in differential and integral calculus, now usually completed not later than the sophomore year. It is with the purpose of supplying such a guide and such a text-book that the editors have associated with themselves various eminent writers, the field to be covered being thus divided among men, each an authority in his particular subject.

The objects thus proposed are most admirable, — the provision of a text which shall aid the student of modern mathematics in laying a good foundation of fundamental principles and elementary development, and which shall broaden his comprehension of the field as a whole, by a fairly rapid

survey of salient features, leaving the development of details to later and more specialized work. Taking the book as a whole, it may be said to realize in good degree the ideal thus set before us. The following subjects are treated, each in a separate chapter, and by the authors as noted: —

I.	The Solution of Equations . . .	Mansfield Merriman.
II.	Determinants	L. G. Weld.
III.	Projective Geometry	G. B. Halsted.
IV.	Hyperbolic Functions	James McMahon.
V.	Harmonic Functions	W. E. Byerly.
VI.	Functions of a Complex Variable	T. S. Fiske.
VII.	Differential Equations	W. W. Johnson.
VIII.	Grassman's Space Analysis . . .	E. W. Hyde.
IX.	Vector Analysis and Quaternions	Alexander Macfarlane.
X.	Probability and Theory of Error .	R. S. Woodward.
XI.	History of Modern Mathematics .	D. E. Smith.

We naturally find in a work of such diverse authorship much difference in style, in mode of treatment, and some unevenness of excellence. Some repetitions are naturally found — hardly avoidable under the circumstances — and as in such cases the subject-matter is approached from different standpoints, they are perhaps advantageous as tending toward breadth of view rather than the reverse. Since the book is intended as a text for students in classical and engineering colleges, it must be considered to some extent, and especially for the latter students, as a compendium of ways and means for the application of advanced higher mathematics to physical and engineering problems. It is in this feature that the work seems most lacking. It was of course no part of the authors' purpose to make a complete mathematical compendium of the branches here treated, and the question is simply as to the amount of recognition this aspect of the book should have received. Some instances may illustrate this point.

Chapter I. on the Solution of Equations takes for discussion designedly only the less well-known methods and points; those not usually treated in elementary texts on the theory of equations. The chapter is thus supplementary to the work usually given in this subject, rather than in any way intended to stand in its place. This is perhaps the more permissible from the fact that the usual course in higher algebra includes the elementary treatment of the theory of equations. For the worker, however, it would be a decided advantage to be able to turn to this chapter and find, in addition to the valuable methods given by the author, a summary or abstract of the more important theorems of the subject as usually pre-

sented. From the limitations of space such matter could not be developed in detail, and would not, of course, answer the purpose of a text for students in the theory of equations. It would be, however, a great convenience to the worker who might wish to freshen his memory as to ways and means rather than in the elementary details, and even without such development would better enable the author to broaden and unify his subject than does the fragmentary treatment given.

Again, in the chapter on Probabilities an admirable development of the subject is given leading up to the theory of least squares, and including much not usually given in the elementary treatment of the subject. The theory of least squares is, however, the most important application of the theory of errors, and it is therefore a disappointment to find the development stop short at this point. The space required would perhaps prevent a detailed treatment, but with the admirable foundation previously given, the salient features of the method might have been briefly sketched and its more important formulæ and methods summarized, much to the convenience of the practical worker.

The most important subject lacking as a whole is that of elliptic functions. It is explained that this was included in the original plan of the book, but that the manuscript could not be obtained in time for publication. It may be hoped that in a later edition this defect may be remedied.

At this point mention may also be made of the subject of higher mechanics, which might well have found a place in a book of this character. It may be said, of course, that higher mechanics is not pure mathematics. Such lines of demarcation, however, are quite artificial, and should not be allowed to outweigh considerations affecting the disciplinary or practical value of the work. For advanced students of applied physics and engineering, such a chapter would have been of great value, while the subject-matter itself would have furnished a rich mine of material for the illustration of various methods and operations treated of in the other chapters of the work.

The wisdom of including Chapter XI., on the History of Modern Mathematics, may also be called in question. Students do not usually study this as a text-book subject, and in any event the development possible in 60 pages is not sufficiently great to give more than the merest outline of the subject. The chapter itself gives in condensed form much general information on its subject-matter, and the present criticism refers simply to the wisdom of its inclusion in a book of this character.

The index for the work, as a whole, is fairly comprehensive, though it bears evidence of having been prepared with some haste, several of the references being incorrect.

A few brief points relating to the various chapters may now be noted.

Chapter I. by one of the editors has been referred to above.

Chapter II., on Determinants, by L. G. Weld, is patterned in condensed form after the same author's well-known text-book on the subject. The elementary development is well presented, with the simpler applications and a number of illustrative problems.

Chapter III., on Projective Geometry, by G. B. Halsted, is well written, though in condensed style. Large use is made of the principles and methods of duality and correlation, the propositions relating to which are presented as usual in parallel columns. The subject is treated synthetically, and a selection of problems and exercises is given. In the latter there might have been included with advantage a greater proportion showing the application of modern higher geometry to other fields of mathematical development.

Chapter IV., on Hyperbolic Functions, by James McMahon, gives an exceedingly well-prepared discussion of the subject. The geometrical character of the functions is clearly shown, and their important properties and relations are logically developed. Numerous applications to physical problems are then given, and the chapter closes with short tables of the numerical values of the functions, conveniently arranged for practical use. Among these is found a table of hyperbolic functions of complex variables, $\sin h\,(x + iy)$ and $\cos h\,(x + iy)$, which is believed to be the first of its kind. By the simple relation between hyperbolic and circular functions ($\sin h\,(i\theta) = i \sin\theta$ and $\cos h\,(i\theta) = \cos\theta$), this table may also be readily used for the circular functions of complex variables.

Chapter V., on Harmonic Functions, by W. E. Byerly, is in the well-known admirable style of the author. The subject is treated synthetically and concretely. That is, the various functions are first developed and discussed in connection with various special physical problems in the solution of which they naturally arise. Their common origin, however, is shown to lie in certain differential equations, and their particular characteristics are shown to depend on the special form of the equation, and on the nature of the coördinates used. The chapter is well supplied with problems for the illustration of the principles developed, and closes with tables of surface zonal harmonics and of Bessel's functions.

Chapter VI., on the Functions of a Complex Variable, by T. S. Fiske, opens with a well-arranged elementary presentation of the subject, with illustrations in conformal representation, and with various applications to problems in hydrodynamics. The more important modern theorems are also given, and the chapter as a whole is well adapted to broaden and deepen the student's conception of the nature and characteristics of measurable quantity in its generalized sense.

Chapter VII., on Differential Equations, by W. W. Johnson, is in the style of the author's well-known text-book on the subject. The elementary portion is clearly presented, and enough of the higher development, including partial differential equations, is given for all purposes, except those of the specialist, for which, of course, the chapter is not intended.

Chapter VIII., on Grassman's Space Analysis, by E. W. Hyde, is in the style of the same author's text-book on directional calculus. The claims of Grassman's method as a space calculus are well presented, though the problems used for illustration are mostly formal rather than practical. Some further remarks on this general subject will be found under the following chapter.

Chapter IX., on Vector Analysis and Quaternions, by Alexander Macfarlane, follows closely along the lines of the same author's previous papers on this subject. Some departures are made from the fundamental definitions and conventions as laid down by Hamilton, and further developed by Tait. These are intended to bring the development of the system into closer agreement with analogies drawn from other branches of mathematics, and to simplify and broaden its application to such branches as a means of research. The chapter is well written and as a whole produces a result consistent with its fundamental basis. The examples are numerous and for the most part practical rather than formal.

In this connection it may not be amiss to note that the progress which space analysis as a means of research has made in the past twenty-five years has been most disappointing. This is doubtless due to many causes. Among them mention may here be appropriately made of the unsettled condition of the questions relating to fundamental convention and to nomenclature. A space analysis must be the expression of its fundamental conventions or definitions, but just what these should be, modern representative writers such as Tait, Heaviside, Gibbs, Macfarlane, and Hathaway are by no means agreed, to say nothing of Hyde as the representative of Grassman's methods. Until a substantial agreement shall be reached on these points and until an approximately constant nomenclature shall be agreed on, the spread of vector methods of analysis, even admitting its superior advantages, will be much hampered.

Of Chapters X. and XI., mention has already been made.

Notwithstanding the few criticisms made, the book as a whole may be cordially commended as of high mathematical and practical value, and so successful an effort at giving a convenient text-book on such a variety of topics in higher mathematics should meet with generous recognition from those interested in these subjects.

W. F. DURAND.

A Primer of Quaternions. By ARTHUR S. HATHAWAY. pp. x + 113. New York, Macmillan & Co., 1896.

This small volume contains four chapters, of which the first treats of steps; the second of rotations, turns, arc-steps; the third of quaternions; and the fourth of equations of the first degree. It presents a good elementary exposition of the method of quaternions, as taught by Hamilton and Tait. The student who masters it will find that he has acquired some real knowledge, not merely additional dexterity in formal manipulations. At the same time it must be said that he will encounter almost all of the difficulties, anomalies, and imperfections which have long obscured the method, and prevented its general acceptance and development.

In recent years it has been maintained, in opposition to Hamilton and Tait, that a vector is a vector, not a right quaternion. Professor Hathaway recognizes the distinction, and proposes to restrict *vector* to mean right quaternion, while using *step* to denote a directed length. This is a proposal which cannot be commended; for writers agree in using the word *vector* to mean a quantity which has linear direction. The confusion is introduced by the false identification on the part of the quaternionists of the vector with the right quaternion. The former has linear direction, the latter angular direction. The plain principle of procedure is to leave "vector" to its established use; and, if necessary, to devise some other word for "right quaternion."

Hamilton laid great stress on his analysis of a quaternion into the sum of a number and a line. From the preface (p. vi) we observe that the author views the whole quaternion as a number. At p. 53 we have the proposition, "The number r rotated through 2 arc q is the number qrq^{-1}." With Hamilton "number" meant a quantity destitute of direction; with the mathematical physicist it means a quantity destitute of physical dimensions; but what meaning are we to attach to it in order that it may be capable of rotation through an angle? A quaternion is not a number in Hamilton's sense of the word, nor is it always a number in the sense understood by the mathematical physicist. The whole resistance in an alternating circuit is the geometrical sum of an ohmic resistance and an inductive resistance; it is a quaternion, but the dimensions are not zero. To explain the quaternion by means of the complex number is to take a step backwards; for Hamilton strove in vain to extend algebra to space until he gave up the time or number idea, and gave the whole a geometrical basis.

In physical science, both the scalar product and the vector product of two vectors are of frequent occurrence; for instance, work is the scalar product of a force and the displacement of its point of application; electro-

motive force per unit of length is the vector product of the velocity of the conductor and the density of the magnetic flux. The Hamiltonian theory attempts to explain AB, the vector product of two vectors, by regarding A as an operator on B. But the explanation given does not explain the scalar product; and, from the physical point of view, it is evident that the two vectors enter symmetrically into the product. The operator and operand theory is entirely inadequate to explain the products which occur in mathematical physics. Professor Hathaway gets over the difficulty by omitting the multiplication of vectors ("steps" in his nomenclature), and treating only of the products of right quaternions.

The order in which Hamilton and Tait write the factors in a product of a number of quaternions does not conform to the order of writing of a sum of angles in trigonometry, or to that of the terms in a series, where the natural order of writing, from left to right, is preserved. Hamilton's innovation arose from the same false analysis of a product into operator and operand. At p. 48, the author explains a product of quaternions thus: "If $rOA = OB$, and $qOB = OC$, and $pOC = OD$, then we have $pqrOA = OD$." This is an explanation which does not explain, for here r, q, and p are not general quaternions, but are restricted to being perpendicular to OA, OB, OC, respectively. When p, q, r, are general quaternions, pqr cannot be explained by any such operation. This is admitted by the author; for, in the next article (p. 49), he says: "The product is, however, independent of whether a step OA can be found or not, such that each factor operates upon the product of the preceding factor; *i.e.* we have, by definition,

$$(a) \quad T(\cdots pqr) = \cdots Tp \cdot Tq \cdot Tr.$$

$$(b) \quad \text{arc}\,(\cdots pqr) = \text{arc}\,r + \text{arc}\,q + \text{arc}\,p + \cdots."$$

Here the author inverts the natural order of writing, except in writing out the sum of the angles, where the established convention in mathematical analysis has been too strong. If it is a matter of definition, as is said above, why is the definition not made to conform with the established practice in trigonometrical analysis? As a matter of fact, when a rotation p acts on a vector OA, the expression is $p^{\frac{1}{2}}(OA)p^{-\frac{1}{2}}$, and no argument can be drawn from the supposed fact that the operator always precedes the operand, for here the operator is written both before and behind.

The author bases the principles of the analysis upon a peculiar mixture of formal laws and mechanical truths. At p. 2, we have an arbitrary definition of the equality of vectors; for it is laid down that all vectors of the same length and direction are equal whatever their point of departure. This is arbitrary; for it is not true mechanically that all vectors of equal magnitude and the same direction are equal whatever their points of appli-

cation. A sum of vectors is said to be both commutative and associative, but no reason is given; on the other hand, a product of quaternions is proved to be associative by means of the highly mechanical consideration of the composition of rotations of a rigid body (p. 39). Surely, all the fundamental rules of the analysis ought to rest on a formal basis, or else they all ought to rest on a geometrical or mechanical basis.

One of the objections to the quaternion method has been its dark symbolic character. In his writings Clifford said that the product of a quaternion and of a vector which is not perpendicular to the axis of the quaternion is an impossible expression without any sense or meaning. In this view he is followed by the author, for at p. 58 it is said that the expression referred to above belongs to that class of products in which the multiplicand does not admit of the operation of the multiplier, as in $\sqrt{2}$ universities, -2 countries, etc. The solution of the mystery is plain; the general product consists of two components, one of which is the resolved part of the vector along the axis of the quaternion, the other being the perpendicular component turned through the proper angle. No doubt, if the projection of the vector on the axis is left out, the product will become mysterious, unless it so happens that there is no component along the axis.

ALEXANDER MACFARLANE.

Alternating Currents and Alternating-Current Machinery. By D. C. JACKSON and J. P. JACKSON. 8vo, pp. 729. The Macmillan Company, 1896.

Some ninety per cent of the ever-growing literature on electrical engineering can be comprised in two classes: theoretical works, written by mathematicians who have never designed, frequently hardly seen, electrical machinery, and thus write books which are as correct and interesting theoretically as useless practically for the engineer, and practical works, whose writers make up for their lack of theoretical knowledge by a generally very limited practical experience. Such latter books may be quite useful as "hints for the dynamo tender," etc., but become disseminators of misinformation when the writer aspires to explain the operation and design of electrical machinery, of which he knows very little himself. Especially vicious this class of books becomes when embodying tables of numerical designing data, either drawn from imagination or by misinterpretation from some antediluvian type of machinery. This state of the literature on electrical engineering is the more deplorable, as all these books are thrust on a public which is not able yet to judge about the worthlessness of the pretended useful information.

It is very gratifying to find in Professors Jackson's work one of the very rare exceptions to the above-mentioned literature, the more as it covers the field of alternating-current apparatus where no large general work of modern nature is in existence, and thus useful information for the student outside of a few of the largest manufacturing concerns is practically inaccessible.

In the first four chapters of the book general matters are discussed, as induction of electromotive forces, resistance, inductance and capacity, wave shapes, armature windings, armature reactions, theoretical methods of investigation, etc. The next four chapters treat of the alternating-current generator, giving amongst others a very good exposition of the problem of parallel operation and of the armature reactions of alternators and their characteristic curves. Chapters IX. to XII. deal in the same thorough and valuable manner with the stationary alternating-current transformer, the next chapter on general polyphase systems. Chapter XIV. treats of alternating-current motors. This chapter, especially the part referring to the induction motor, is decidedly weak compared with the other chapters of the book. The last chapter discusses polyphase transformations. A number of appendices attached to the book give Fourier's series and characteristic features of alternating-current curves, oscillating discharges, and electrical resonance. A very complete index is attached, increasing essentially the usefulness of the book.

While undoubtedly by far the best work on the state of the art of alternating-current engineering and more in touch with modern practice than any other general publication, Professors Jackson's book, nevertheless, cannot quite represent the present state of the art. The development of alternating-current work has practically been monopolized by two or three large manufacturing concerns and an enormous amount of investigation carried on by them. What little of the knowledge thus accumulated is accessible to the general public is due to occasional publications of insiders of these concerns, and to tests made by our universities. Especially in view of the insufficient protection of our patent law, insiders of these manufacturing concerns are necessarily loath to disclose much of a knowledge which has cost enormous sums to accumulate, while tests made by our universities are at the best limited in extent, and frequently made on apparatus which no longer represents the most modern state of the art. This does not distract from the value of Professors Jackson's book, but rather increases it, since, as stated above, the book is in better touch with the most recent development of engineering than any other publication, but it is necessary to mention it so that the student may not be led to believe that he can design modern alternating-current machinery after the study of this work.

Especially noticeable is this feature in the chapter on the induction motor, as the apparatus on which development has progressed most rapidly in the last years. Here the fundamental importance of the internal self induction of the motor is not recognized at all, and in consequence thereof the conclusions regarding the effect of the frequency, and the change from polyphase to single phase are erroneous. For a given size and speed a certain frequency gives the best all-around induction motor. At lower frequencies the efficiency and speed regulation are impaired, at higher frequencies the power factor drops and the current at light load rises. At the same magnetic density a single phase motor gives only somewhat less than half the output of the same motor as a polyphase induction motor, at nearly the same loss, and thus when raising the density so as to get in the single phase induction motor about three-fourths the output as on a polyphase circuit, the efficiency is considerably lower, the exciting current higher, and thus the power factor lower, but the speed regulation better. At the same magnetic density the loss in the iron is shown by tests to be the same in the single phase induction motor as in the polyphase motor. Thus the magnetism in the secondary of the single phase motor is not alternating with the primary frequency but only with the frequency of slip, the same as in the polyphase motor. The representation of the magnetic condition of the single phase motor by two magnetic fields rotating in opposite direction does not represent the actual facts, but would require at synchronism a current equal to the exciting current of the polyphase motor at half voltage plus the primary current of the polyphase motor when driven backwards at synchronous speed and at half voltage, that is, a current more than half the current at standstill. Tests, however, show the current in some single phase induction motors when running light to be less than twenty per cent of the current when standing still. The magnetic field of the single phase induction motor consists of a field of approximately constant intensity — reduced at larger current input only by drop of voltage due to the internal impedance — with a second magnetic field superimposed thereon in quadrature in space and in phase, and approximately proportional to the speed of the motor.

The electrical literature contains a number of erroneous statements made years ago, which have passed from publication to publication without being challenged. Very few of these have entered Professors Jackson's book. One I noticed: the statement of the absence of magnetic leakage in the Mordey type of alternators. If magnetic leakage means anything, it means those lines of force which pass through the magnetic field without contributing to the terminal voltage of the machine. It is thus obvious that in the Mordey type of alternator where the induction is due to the difference of magnetic flux in front of the pole faces and between the pole faces, every

line of magnetic force which passes through the space between the pole faces represents two lines of magnetic leakage flux. It is this very type of machine which, other things being equal, has a larger magnetic leakage than the ordinary type of alternator. Thus close regulation in the Mordey type of alternator is not due to the absence of magnetic leakage but to the enormous magnetizing force available by the use of a single magnetizing coil.

Very valuable is the explanation and discussion of the Fourier's series in the first appendix of Professors Jackson's book, since this series has proven of great importance in the theoretical investigation of alternating-current phenomena, and is still very scantily treated in most text-books.

C. P. STEINMETZ.

Transformers for Single and Multiphase Currents. Translated from the German by the author. By GISBERT KAPP. 8vo, pp. 241. London, Whittaker & Co., 1896.

The literature of alternating currents is a product of the last six or eight years, and books dealing solely with this subject now number about a score. A few of these, consisting chiefly of mediocre cuts and meager description, have been written to catch the eye of the so-called "general public," or to delude the dynamo attendant into the belief that after a few weeks of evening study he can undertake the design of systems for long-distance power transmission; but with these few exceptions the books on the subject have been contributions, each independent and filling a field of its own. At the beginning of the year 1896, publishers' announcements called attention to the fact that the year was to bring forth some half dozen works[1] on alternating currents and the alternating-current transformer, the authors — German, French, British, and American — being writers of reputation. The year's addition to the subject has been a most valuable one; with one exception — a volume[2] more in the nature of a general compendium than a methodical treatise — each work has treated the subject along clearly defined lines, the distinctness of treatment being as marked as though each author were aware of the other's work and carefully avoided trespass. The reader thus has the advantage of being able to refer to several books treating the subject from different standpoints to supplement the book selected for chief study.

Although the advantage of individuality of treatment is thus obvious, it is to be noted that the criticism of books of this class is commonly (in

[1] The belated book of Mr. Steinmetz (1897) is included in this category.

[2] Reviewed in the PHYSICAL REVIEW, November-December, 1896.

substance) that A's book is worthless because not written from the standpoint of B, and (by another critic) that B's book is valueless because not written for the readers reached by A. Both books may be excellent, each for its own class of readers; but they are condemned because written from a different point of view from that of the critic, and because they omit that which it was never intended they should contain.[1] The choice of a book depends upon by whom it is to be read, and for what purpose.

For a reader desiring a clearly written, accurate, and readable discussion of the transformer, touching all essential points without dwelling too much upon any particular phase of the subject or upon abstruse theory, the present book by Mr. Kapp is admirably suited. The book contains much substantial material, either for the specialist or the general reader. No book on the subject is written in such a pleasing style; Mr. Kapp keeps the attention of his reader and carries him through much general explanation and accurate detail with ease. If one should seek to learn, however, the theory of alternating currents solely from its pages, he needs must be disappointed; for he would encounter gaps of such magnitude that he could scarcely develop a theory other than incomplete and fragmentary. As stated in the preface, the book is written particularly for those interested in the design of transformers. In the author's words, "I am well aware that through the omission of many theoretical considerations which were not immediately pertinent to the practical object I had in view, my book, considered as a scientific treatise on transformers, is incomplete." He says, further, that his book is written for engineers, whose object must always be to obtain a maximum of practical success with a minimum expenditure of mental labor. In this the author has succeeded.

Although written with this object, the book will prove a valuable source of information to any who may read it. It is not suited for use as a college text-book. Being written by one familiar with the theoretical side of the subject as well as the practical, it is an admirable book to be read as a supplement to treatises devoted more particularly to the principles of the transformer.

Chapter I. deals with fundamental principles and magnetic leakage. The author immediately enters into his subject with commendable directness, general remarks and preamble being omitted. It is not customary and scarcely advisable in treatises of this kind to introduce so early the subject of magnetic leakage. Mr. Kapp introduces it on the very first page, and its discussion constitutes a large portion of the first chapter. Mr. Kapp uses at once the sine assumption (p. 10). Despite the past outcry by so-called practical men, theorists and practical engineers alike have come to

[1] A point well brought out by Professor Trowbridge in a letter in The Electrician (Dec. 25, 1896), and in Nature (Jan. 14, 1897).

recognize the validity of many conclusions based upon an assumed harmonic electromotive force or current. The discussion of the fundamental equations of the transformer is introduced by the following unqualified statements: "*We also assume that the primary current is obtained from an alternator, the electromotive force of which follows a true sine law. This is not always, and indeed very seldom, the case; but it will be shown later on that the equations obtained under these assumptions remain applicable in all cases occurring in practice.*"

The arithmetical proof (p. 15) that the maximum value of a sine function is equal to $\sqrt{2}$ times the square root of the mean square of the instantaneous values is direct and simple. It is convincing to those whose knowledge of calculus does not allow them to follow the proof commonly given.

The losses in transformers are discussed in Chapter II., and the influence of the shape of the electromotive force curve on hysteresis loss. The six pages treating of the latter are very clear, and much more to the point than the wordy discussion of the subject recently published in the technical press. The chapter closes with a discussion of core and shell transformers with parts of different proportions, making obvious the advantages of the generally used modern type.

Chapter III. contains a discussion of the proportion of transformers, and the effect of alterations, of linear dimensions, much of which the author has published before. It is rewritten, and made clearer and more complete. Attention is particularly called to the necessity of providing proper cooling surface in the design of a transformer.

Chapter IV. outlines, in the usual way, familiar principles of alternating currents, the vector addition of currents and electromotive forces, and the determination of the power of an alternating current. The chapter closes with several good pages on the calculation of the no-load current and on the influence of joints in the magnetic circuit.

Chapter V. contains much of value to the designer, discussing at length the best distribution of copper and distribution of losses. Following this, a long chapter is devoted to clock diagrams and the graphic determination of drop. The chapter contains much good material, but a better arrangement of the diagrams would make the work easier to follow.

The next chapter gives, in brief, the usual methods of measuring the power of transformers, dynamometer, wattmeter, three-voltmeter and three-ammeter methods, and methods for testing iron. In a practical book of this kind, more stress might be laid upon the fact that the three-voltmeter and the three-ammeter methods are scarcely suited for common practice.

Chapter VIII. is an eminently practical chapter, discussing safety appliances, substations, house transformers, choking coils, compensating coils,

and other subjects relating to distribution. The book closes with a chapter on illustrated descriptions of some transformers. This chapter consists of cuts, chiefly of continental transformers, with brief descriptions.

As we finish the book, we are led to wonder at the title, "Transformers for Single and *Multiphase* Currents." One might conclude that "Multiphase" was introduced into the title merely to give it a fine sound, or to gain the attention, perhaps, of some would-be reader. In vain we turn the pages of the book to find more than a casual mention, and, perhaps, two or three cuts of multiphase transformers. "Multiphase," moreover, is considered by some a hybrid substitute for "polyphase."

As exceptions to the usual lucidity of the book, we may note p. 20, where the discussion of the relation between eddy current and thickness of lamination is not clear upon first reading. On p. 53, the scale employed in plotting the curves showing the rise of temperature in terms of the area allowed per watt loss is not obvious, the omission of the scale being unfortunate as the curves are intended for practical use. In nomenclature, we may take exception to W for resistance, and $\sim$ as a symbol for frequency. The latter is scarcely suited to represent a *quantity* in an equation, no more so than $\ast$, !, or $\frown$. Some may object to "effective" for "virtual" (p. 16), to "ampèremeter" for "ammeter" (p. 161), to "an electricity works" (p. 160), to "so-called current heat" (p. 19) as indicating the loss due to ohmic resistance, and to "E.M. forces" for *E.M.F.'s*, or, better still, *electromotive forces*.

Minor typographical errors, for the most part obvious, are all too frequent: thus one of limit of an integral is omitted (p. 16; p. 149); we note v for o (p. 12); *cos* for cos^2 (p. 14); *sinnsoidal* for *sinusoidal* (p. 17); *less* for *loss* (p. 21); *coil* for *oil* (p. 58); *io* for i_0 (p. 78); M for μ (p. 80); $1_1 78$ for 1.78 (pp. 80 and 81); *forces* for *fuses* (p. 177).

FREDERICK BEDELL.

Méthode et Principes des Sciences Naturelles. TH. FUNCK BRENTANO. pp. 137. Paris, L. Bataille & C^ie^, 1896.

The purpose of the writer is evidently twofold; first, to urge upon students of the natural sciences a more logical habit in discussion of scientific principles, and, secondly, to correlate the forces of nature under one pleasing hypothesis of the author's invention. To attain these ends the book is divided into two parts, one, treating of *Method*, having for object the mental development of the reader to a point which will render possible the unreserved acceptance of the second part, the *Principles*. The difficulty and futility of classifications, the relations of the various divisions of science and of thought, experimentation, abstraction, analysis, synthesis,

induction and deduction, are all duly discussed. Then follows the hypothesis, with a distinct Cartesian ring, the two realities, form and movement, and the explanation thereby of all chemical and physical phenomena. Affinity, attraction, electricity, light, and heat are all simply and easily explained by peculiarities of form of the atomic and molecular units with the resulting variation in ethereal currents.

An appendix discusses briefly the photography of the invisible, claiming for the *rayons x* a niche in the grand scheme already presented. Light is described as a result of two sets of vibrations, neither alone visible, the primary being those imparted to the ether by the movement of elementary forms in passing from the free state to a molecular combination, and the secondary, those due to the inter-molecular movements of the atoms. The "X ray" is to be considered as an exaggerated development of the latter, the secondary, vibrations.

CHARLES PLATT.

PHILADELPHIA, PA.

Motive Power and Gearing for Electrical Machinery. By E. TREMLETT CARTER. 8vo, pp. xxi+620. New York, D. van Nostrand Company; London, The Electrician Publishing Company; 1896.

The author of this somewhat extensive and very timely work has brought together, in a systematic and logical shape, a large amount of valuable scientific matter and practical knowledge of special value to the engineer engaged in the design, the construction, or the operation of electric light or power "plants." As he states in his preface, he recognizes the facts that electrical engineering is a branch of mechanical engineering of great and growing importance, and that the practitioner must, to succeed to-day, possess a good knowledge both of the science and of the practice of the art. "Just as the marine or locomotive engineers have to make a special study of the steam engine in its relation to marine or railroad practice, so must the electrical engineer study, from his own point of view, the principles and practice of motive power and gearing for electrical machinery." He includes in his field of study gas engines and hydraulic motors, as well as the steam engine. The presentation of the results of trials of various forms of motor, and that of the details of station and "plant" construction constitute extensive and important parts of his work.

In Part I. are given the fundamental principles of mechanical science involved in this branch of engineering, and definitions of scientific and technical terms now accepted by the practitioner and the man of science dealing with such subjects. The problem in hand is stated thus: "What are the best provisions for utilizing the available energy for the performance of useful work and the production of a paying revenue?" The last require-

ment is one which, though vitally important, is rarely considered in works of this kind; and this may be cited as one of the points of novelty, as well as of excellence, of the book. All such propositions are printed in heavy-faced type, — another good point.

Among the principles thus specially impressed upon the mind of the reader are the following: —

"The most economical design is that in which the annual interest on the cost of any small improvement in the efficiency of the plant is equal to the saving of expenses which would result annually from that improvement being adopted."

"In any arrangement of mechanical or other parts for the transmission of energy, the energy entering the arrangement at an irregular or fluctuating rate, and leaving it at a regular rate, or *vice versa*, the successive parts will be subjected to the least straining action, and will be required to transmit the least power, when the store or reservoir for transforming the rate from the one value to the other is placed at the least possible distance from the actual seat of the fluctuation."

In this portion of the book, the economics of the engineer's problem are studied in various aspects and the principle clearly shown that his work is, in fact, throughout that of securing a stated result at minimum "running cost." The discussion of the bearing of the "factor of idleness" upon this "commercial efficiency," as the writer has called it, is one of the most suggestive parts of this introductory matter.

Part II. is devoted to the steam engine, including its fuels, the thermodynamics of the case, the structure, theory, and action of the engine and of its boiler. Engine testing and the Manchester Steam Users' Association's instructions, the methods of choice of type of engine and boiler and the best system of subdivision of the plant are given considerable space. In the thermodynamic discussion, the author takes $J = 772.55$, and $\gamma = 1.408$. This section is particularly well condensed. It appears to be generally accurate as well; but the author follows Rankine in making, for adiabatic expansion of initially dry steam, $pv^{1.111} = \text{constant}$, instead of adopting the better established index 1.135. The expedients of superheating and steam jacketing are very justly spoken of as thermodynamic errors, but "necessary evils."

Part III. is devoted to gas and oil engines, their fuels, the thermodynamics of this class of motors, and their special adaptation to the development of power in electric stations. The best and the most familiar of recent types are described, and their performance, on trial and in use, discussed. Part IV. treats of water power and hydraulic motors in condensed and well-arranged form. Part V. discusses gearing and Part VI. the arrangement and operation of stations, including, to an exceptional and commendable extent, the questions of costs and profits.

The book is well made, although the illustrations, which are numerous and well chosen, are not fully up to the standard of the best book-making of the time. A good index, a comparatively rare appendix to an English technical publication, completes the book, which will be likely to find many readers, and to be well worth its price to the practitioner in this field.

R. H. THURSTON.

The Elements of Electrochemistry. By MAX LE BLANC. Translated by W. R. WHITNEY. 8vo, pp. 282. The Macmillan Company, 1896.

The German edition of this work has been reviewed at length in the PHYSICAL REVIEW, Vol. IV., p. 268. The translator has followed the original work as nearly as practicable; the English edition is well written and its mechanical make-up is good.

Mathematical Papers read at the International Mathematical Congress, Chicago, 1893. Edited by the Committee of the Congress, E. HASTINGS MOORE, OSKAR BOLZA, HEINRICH MASCHKE, HENRY S. WHITE. pp. xvi.+411. New York, Macmillan & Co., for the American Mathematical Society, 1896. (*Received.*)

Laboratory Manual of Inorganic Chemistry. By RUFUS P. WILLIAMS. pp. xxii and 99. Boston, Ginn & Company, 1896. (*Received.*)

Chemistry at a Glance: A Study in Molecular Architecture. By HERBERT B. TUTTLE. 8vo, pp. 59. New York, 1896. (*Received.*)

Inorganic Chemical Preparations. By FRANK HALL THORP. 8vo, pp. 238. Boston, Ginn & Company, 1896. (*Received.*)

The American Annual of Photography and Photographic Times Almanac for 1897. 8vo, pp. 370. New York, The Scovill & Adams Company. (*Received.*)

Problems and Questions in Physics. By CHARLES P. MATTHEWS and JOHN SHEARER. 8vo, pp. 247. The Macmillan Company, New York, 1897. (*Received.*)